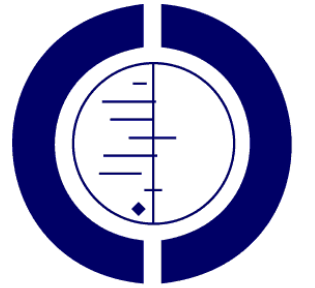




# Statistical considerations in indirect comparisons and network meta-analysis

Saïd Business School, Oxford, UK

March 18-19, 2013



THE COCHRANE  
COLLABORATION®

Handout S2-L

# Review of standard meta-analysis methods & Introduction to indirect comparison

Tianjing Li

## Handbook Chapter 9 in a nutshell

analyses (91) analysis (67) characteristics (46) clinical (53)  
cochrane (34) combined (32) comparisons (33) consider (33) continuous (42) control (35)  
data (161) dichotomous (45) difference (156)  
effect (244) error (41) estimate (45) events (114)  
example (65) fixed-effect (35) heterogeneity (107)  
important (36) intervention (159) investigate (56) log (30)  
measures (44) meta-analyses (34) meta-analysis (168)  
meta-regression (31) methods (75) number (60) observed (43)  
odds (106) outcome (74) participants (51) random-  
effects (48) ratio (168) results (104) review (103)  
revman (37) risk (183) scales (31) section (114)  
standard (86) statistical (110)  
studies (218) subgroup (52) summary (55) systematic (34)  
treatment (37) trials (48) value (31)

# Fixed-effect Model

## Fixed-effect Model Assumes a Common (“True”) Effect Size

Under the fixed-effect model, we assume

- All studies share a common (“true”) effect size ( $\theta$ )
- All factors that could influence the effect size are the same in all studies
- All observed variation reflects sampling error
- Study weights are assigned proportional to the inverse of within studies variance

# Fixed-effect Model – True Effects and Sampling Error

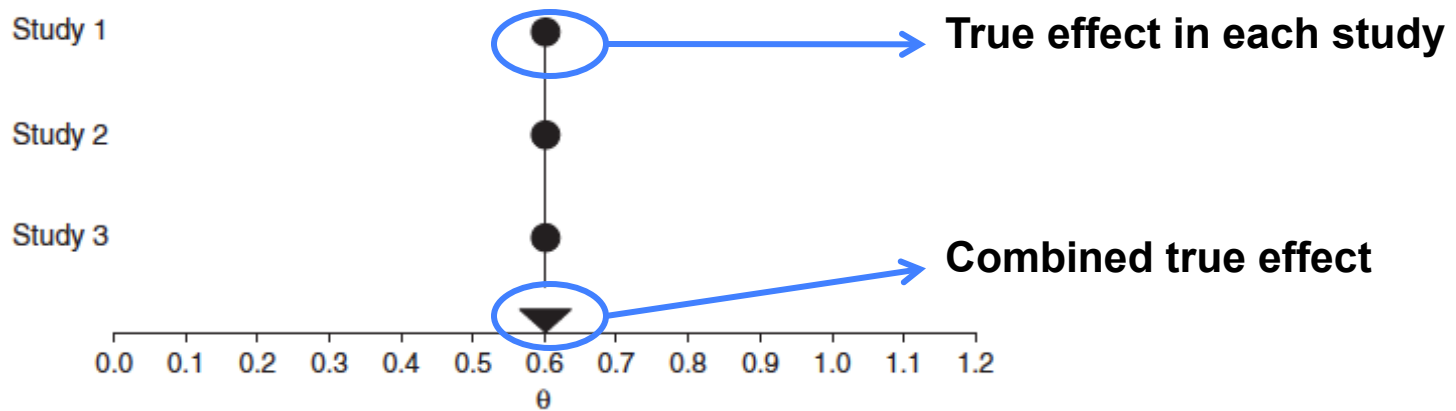


Figure 11.1 Fixed-effect model – true effects.

The observed effect size varies from one study to the next only because of the random errors inherent in each study.

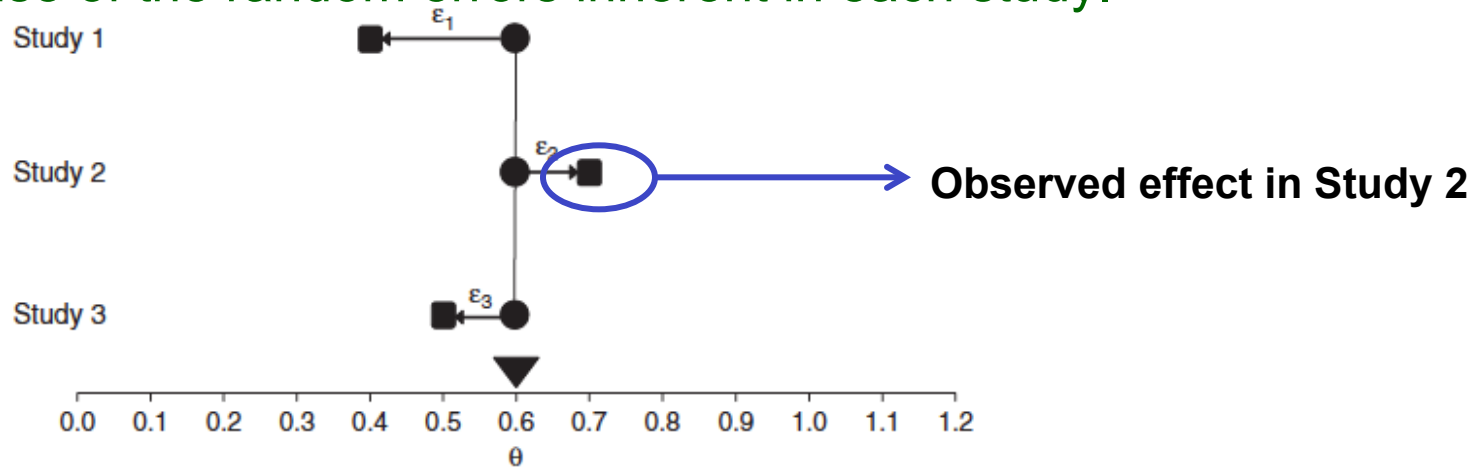


Figure 11.2 Fixed-effect model – true effects and sampling error.

# Fixed-effect Model – True Effects and Sampling Error

The observed effect size varies from one study to the next only because of the random error inherent in each study.

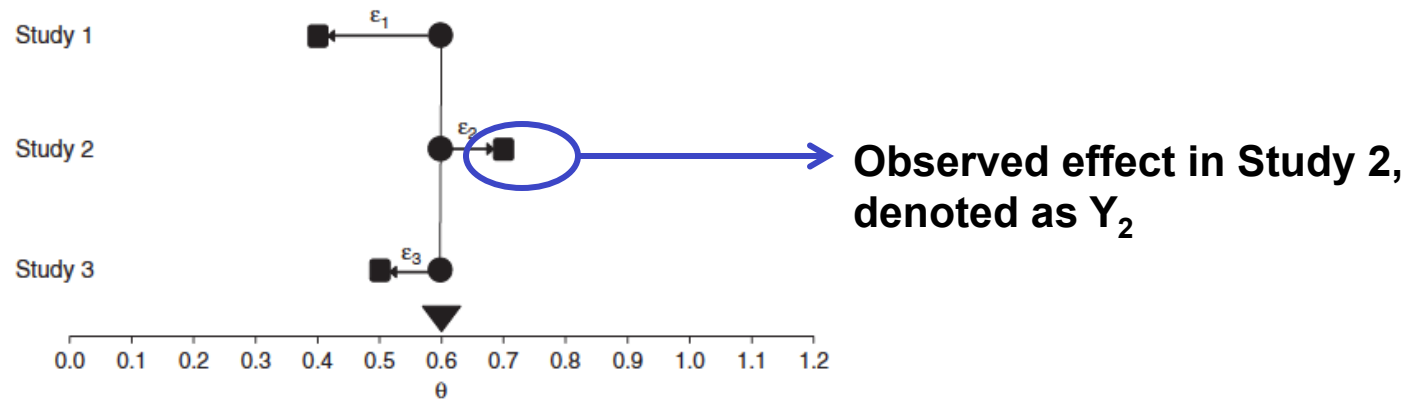


Figure 11.2 Fixed-effect model – true effects and sampling error.

The sampling error ( $\epsilon_i$ ) is -0.20, 0.10, and -0.10 respectively in Study 1, 2, and 3.

$$Y_1 = 0.60 - 0.20 = 0.40$$

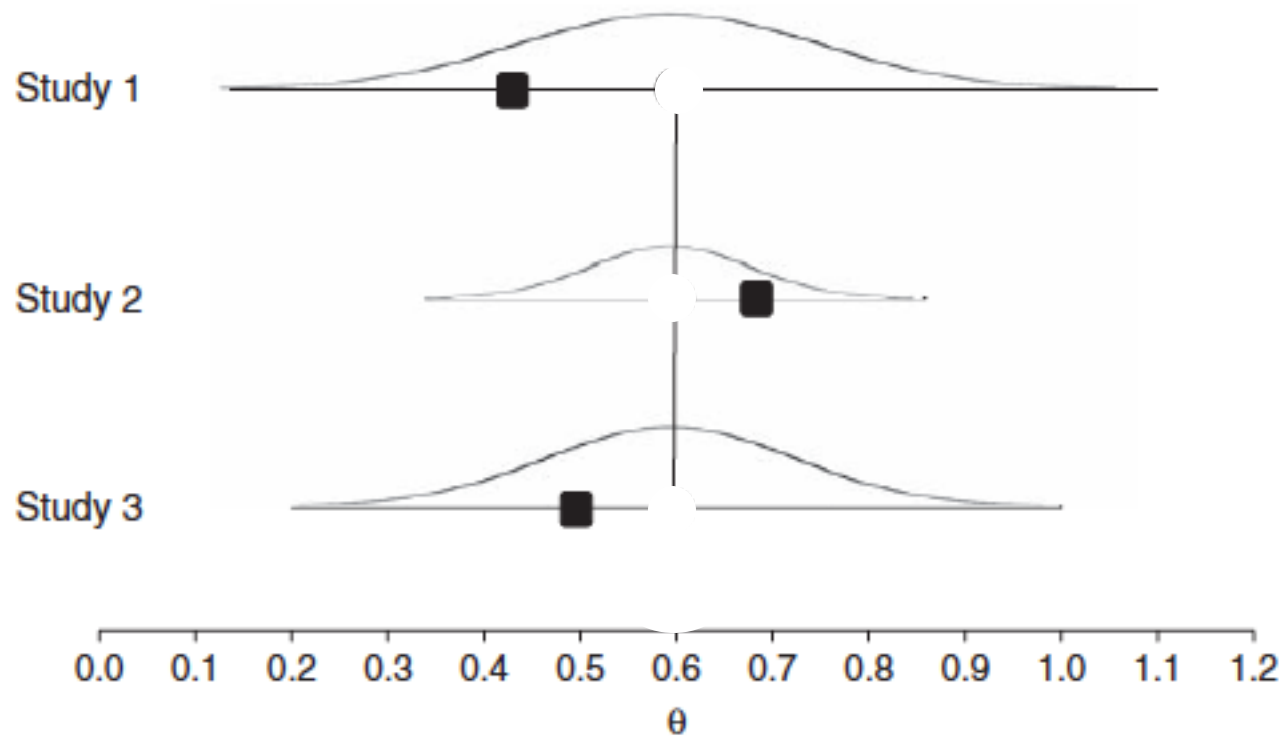
$$Y_2 = 0.60 + 0.10 = 0.70$$

$$Y_3 = 0.60 - 0.10 = 0.50$$

**More generally, the observed effect  $Y_i$  for any study is given by the population mean plus the sampling error in that study.**

$$Y_i = \theta + \epsilon_i$$

- One source of variance (ie, random errors inherent in the study)
- The width of the normal curve is based on the variance in that study.



**Figure 11.3** Fixed-effect model – distribution of sampling error.



# Performing a Fixed-effect Meta-analysis

Start with the observed effects and try to estimate the population effect through computing a weighted mean.

- **Weight** assigned to each study in a *fixed-effect* meta-analysis is

$$W_i = \frac{1}{V_{Y_i}} \quad V_{y_i} \text{ is the within study variance for study } i$$

- **Weighted mean ( $M$ )** is computed as

$$M = \frac{\sum Y_i W_i}{\sum W_i}$$

- **Variance of the summary effect ( $V_M$ )** is estimated as

$$V_M = \frac{1}{\sum W_i}$$

- **Standard error of the summary effect ( $SE_M$ )** is estimated as

$$SE_M = \sqrt{V_M}$$

# Random-effects Model

## Is the Assumption Underlying a FE Model Plausible?

- Fixed-effect models assume that the studies are identical and the true effect size is exactly the same in all studies.

### In reality...

- Studies usually differ in the mix of participants and in the implementations of interventions etc.
- There may be different effect sizes underlying different studies.

## Is the Assumption Underlying a FE Model Plausible?

- For example, the magnitude of the impact of an educational intervention might vary depending the class size, the age, and other factors, which are likely to vary from study to study.
- We may or may not know for sure whether these characteristics are actually related to the size of effect.
- Nevertheless, logic suggests that such factors do exist and will lead to variations in the magnitude of the effect.

## Heterogeneity and a Distribution of True Effects

- Careful qualitative synthesis of the data indicates that clinical and methodological diversities usually exist and may lead to variations in the magnitude of the effect.
- Instead of assuming there is one common true effect (as in a fixed-effect model), shall we assume that there is a distribution of true effects?

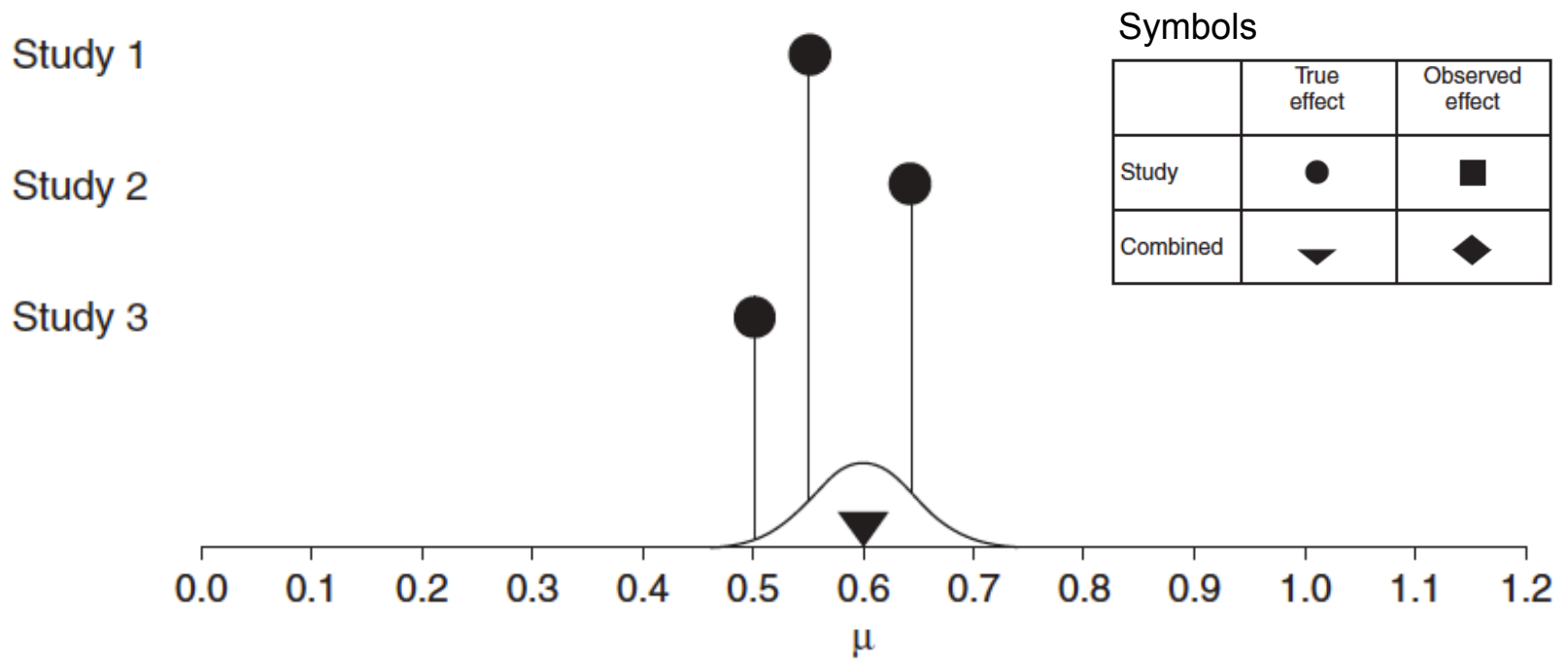


Figure 12.2 Random-effects model – true effects.

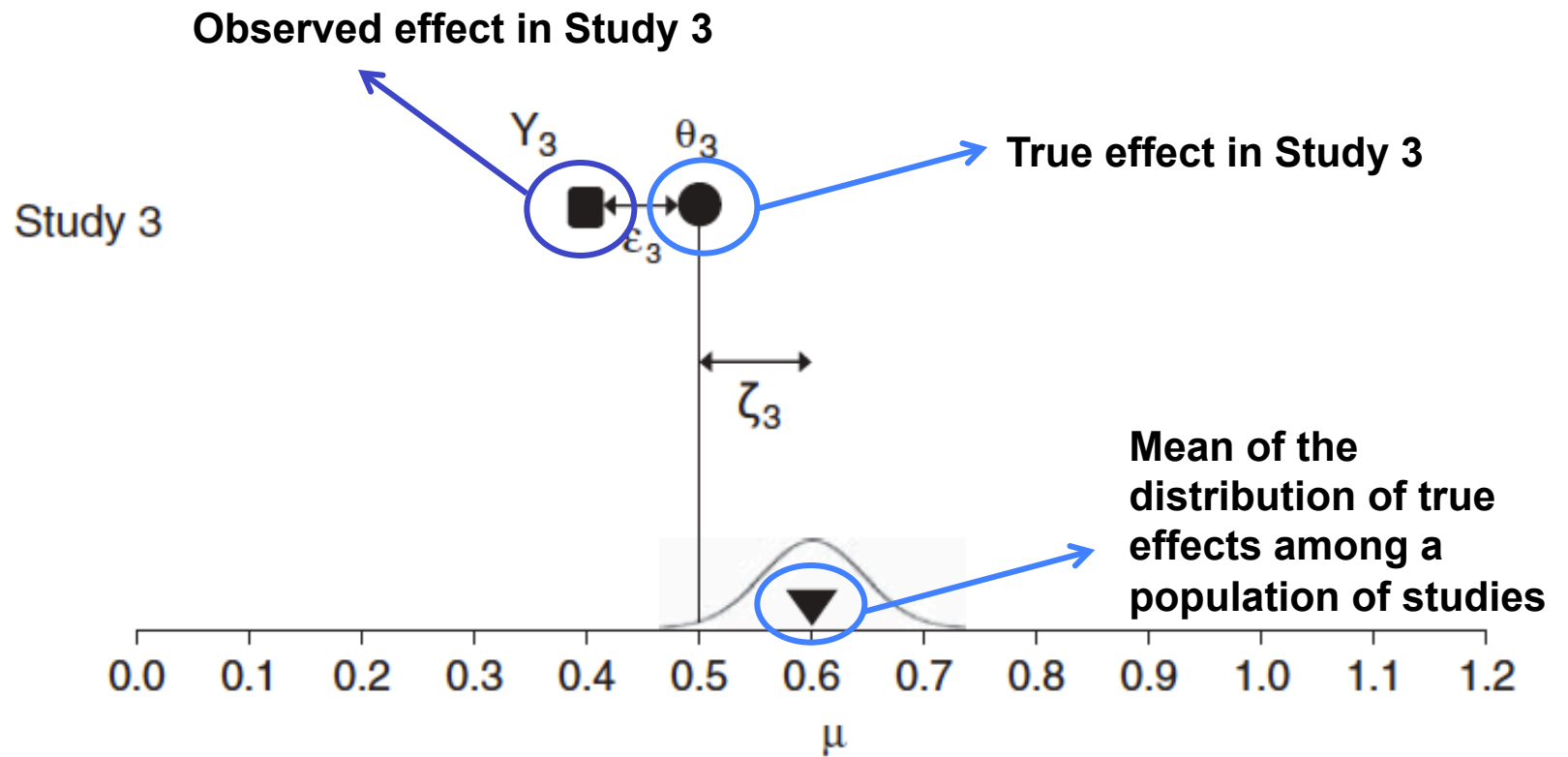
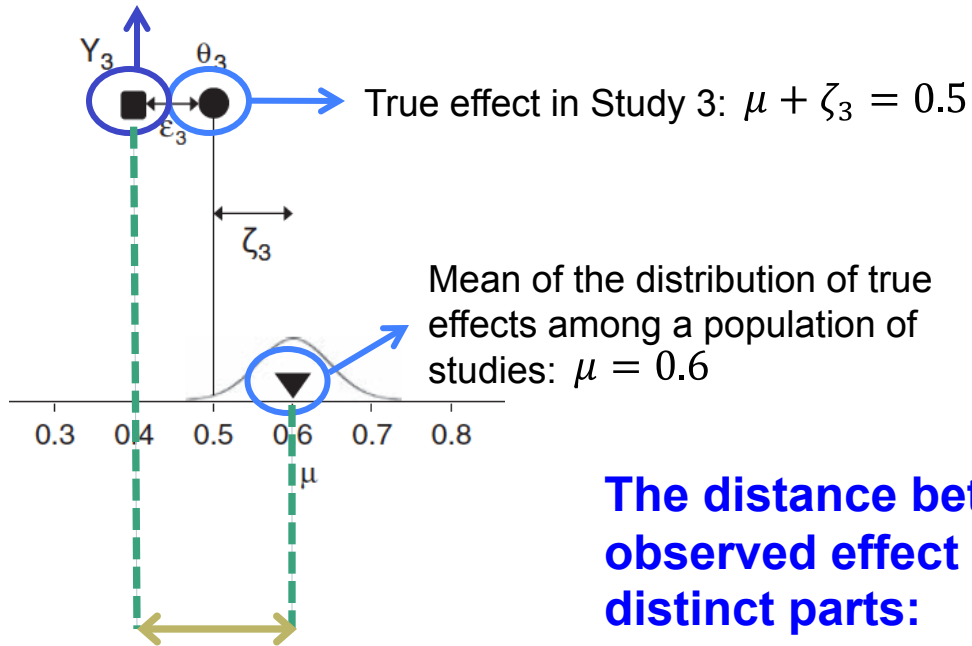


Figure 12.3 Random-effects model – true and observed effect in one study.

Observed effect in Study 3:  $Y_3 = \mu + \zeta_3 + \varepsilon_3 = 0.4$



Symbols

	True effect	Observed effect
Study	●	■
Combined	▼	◆

**The distance between the overall mean and the observed effect in any given study consists of two distinct parts:**

- True variation in effect sizes ( $\zeta_j$ )
- Sampling error ( $\varepsilon_j$ )

**More generally, the observed effect  $Y_i$  for any study is given by the grand mean, the deviation of the study's true effect from the grand mean, and the sampling error in that study.**

$$Y_i = \mu + \zeta_i + \varepsilon_i$$



# Random-effects Model – Two Sources of Variance

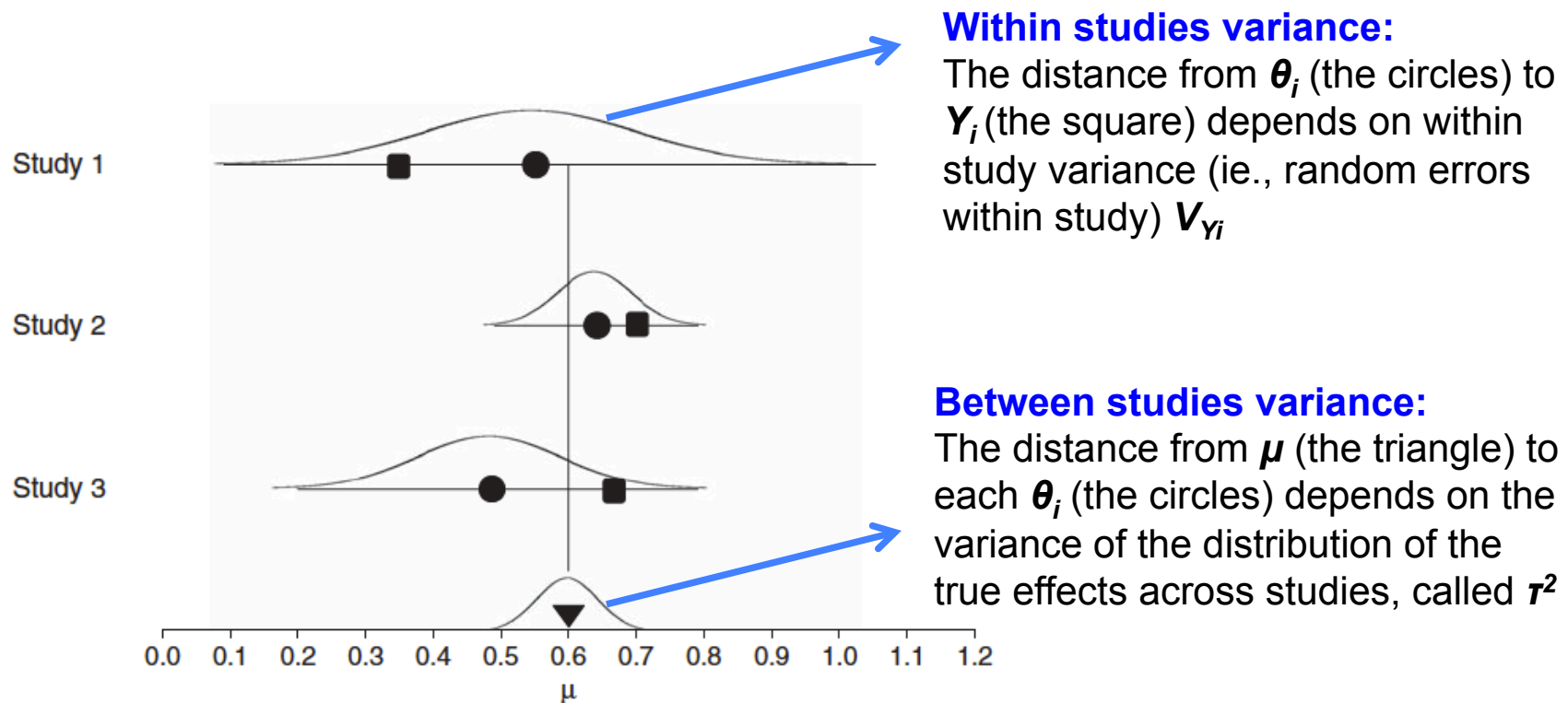


Figure 12.4 Random-effects model – between-study and within-study variance.

# Fixed-effect                      Versus                      Random-effects

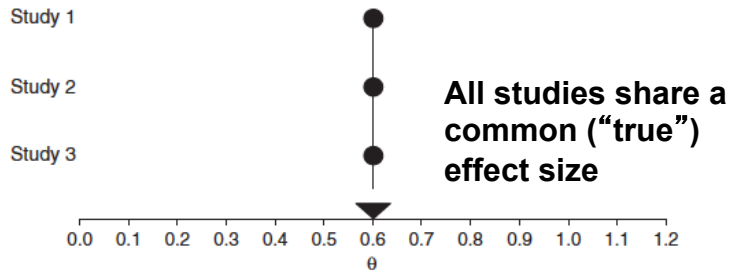


Figure 11.1 Fixed-effect model – true effects.

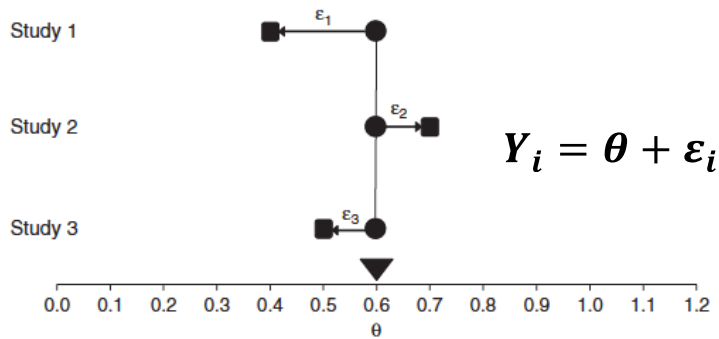


Figure 11.2 Fixed-effect model – true effects and sampling error.

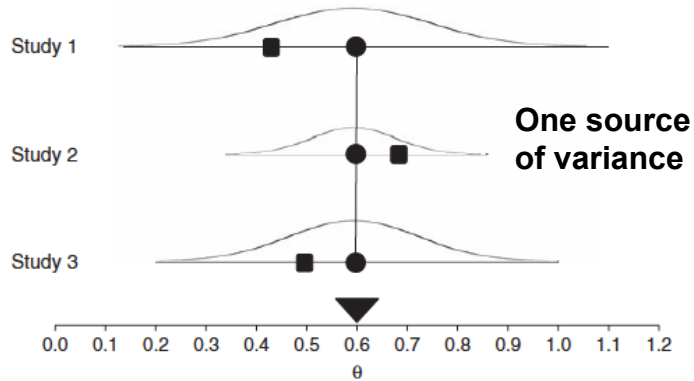


Figure 11.3 Fixed-effect model – distribution of sampling error.

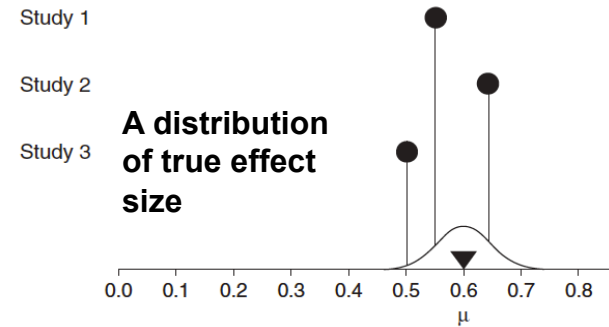


Figure 12.2 Random-effects model – true effects.

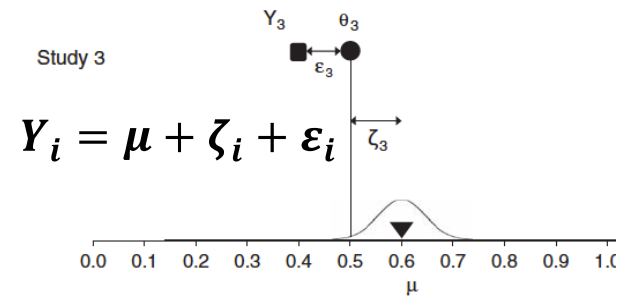


Figure 12.3 Random-effects model – true and observed effect in one study.

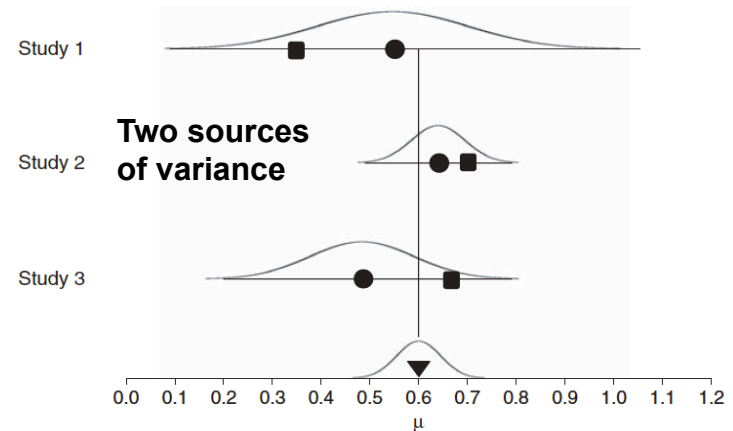


Figure 12.4 Random-effects model – between-study and within-study variance.

# Performing a Random-effects Meta-analysis

## Analysis goal

- In an actual meta-analysis, we start with the observed effects and try to estimate the population effect.
- Goal is to use the collection of  $Y_i$  to estimate the overall mean  $\mu$ .

## How?

- Overall mean is calculated as a weighted average; the weight assigned to each study is the inverse of that study's variance.
- The variance now includes the ***within*** studies variance plus the estimate of the ***between*** studies variance.

# Performing a Random-effects Meta-analysis

Start with the observed effects and try to estimate the population effect through computing a weighted mean.

- **Weight** assigned to each study in a *random-effects* meta-analysis is

$$W_i^* = \frac{1}{V_{Y_i}^*}$$

$V_{Y_i}^*$  is the within studies variance for study  $i$  plus the estimate of between studies variance  $T^2$

$$V_{Y_i}^* = V_{Y_i} + T^2$$

- **Weighted mean ( $M^*$ ):**  $M^* = \frac{\sum Y_i W_i^*}{\sum W_i^*}$
- **Variance of the summary effect ( $V_{M^*}$ ):**  $V_{M^*} = \frac{1}{\sum W_i^*}$
- **Standard error of the summary effect ( $SE_{M^*}$ ):**  $SE_{M^*} = \sqrt{V_{M^*}}$

$\tau^2$  is the between studies variance. Its estimate is denoted as  $T^2$ .

### DerSimonian and Laird Method (Method of Moment)

$$T^2 = \frac{Q - df}{C}$$

$$Q = \sum_{i=1}^k W_i (Y_i - M^*)^2 = \sum_{i=1}^k W_i Y_i^2 - \frac{(\sum_{i=1}^k W_i Y_i)^2}{\sum_{i=1}^k W_i}$$

$$df = k - 1$$

$$C = \sum W_i - \frac{\sum W_i^2}{\sum W_i}$$

**A caveat:** if the number of studies is small, then the estimate of the  $\tau^2$  will have poor precision.

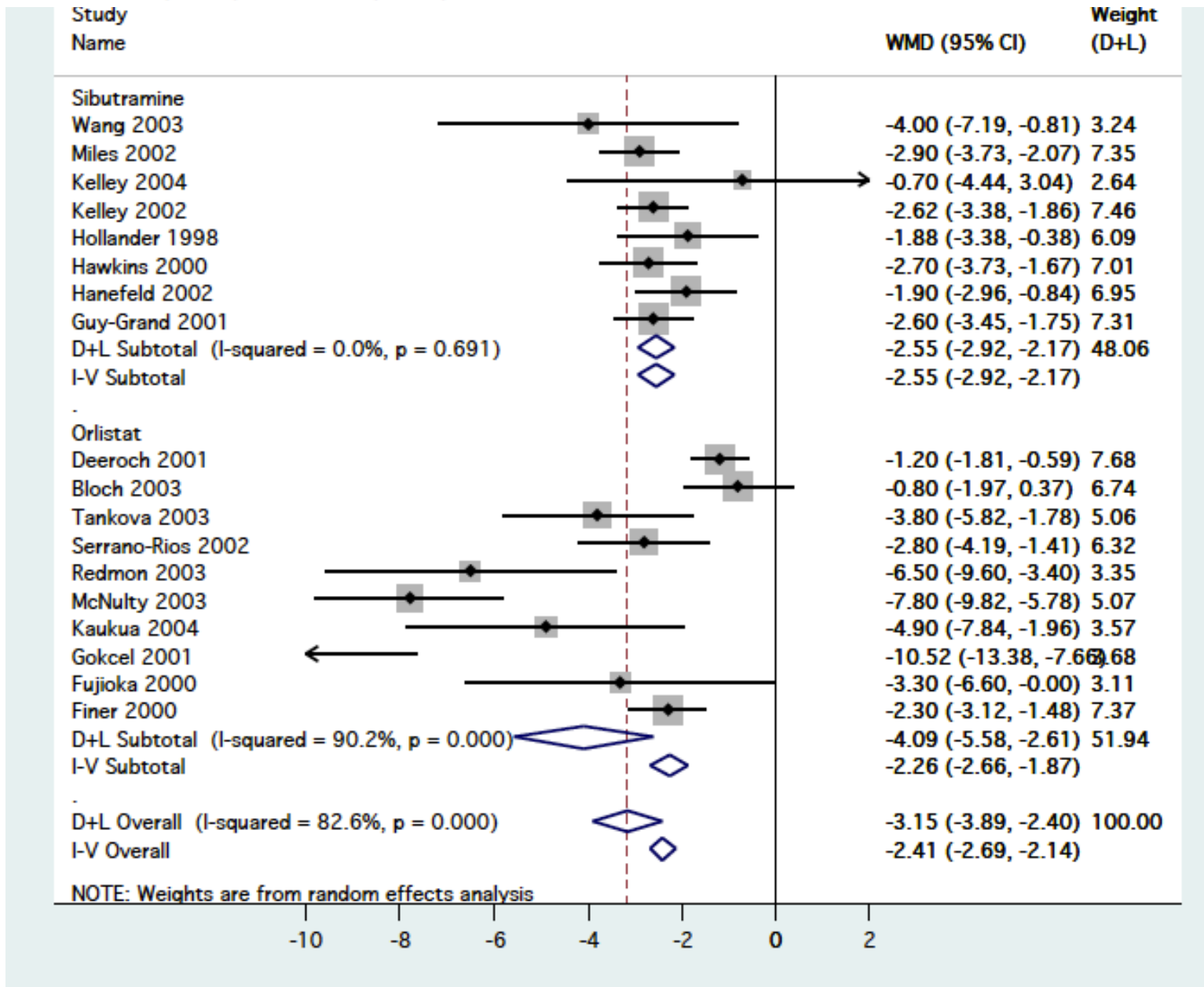
## Pharmacotherapy for Weight Loss in Adults with Type 2 Diabetes

Drug Name	Variable name Study Name	Weigh Loss Drug			Placebo		
		Total n_treat	Mean mean_treat	SD† sd_treat	Total n_plc	Mean mean_plc	SD sd_plc
Sibutramine	Finer 2000	43	-2.4	2.03	40	-0.1	1.77
	Fujioka 2000	60	-3.7	9.22	61	-0.4	9.29
	Gokcel 2001	29	-9.61	7.38	25	0.91	2.47
	Kaukua 2004	102	-7.3	10.71	108	-2.4	11.02
	McNulty 2003	49	-8.0	6.3	46	-0.2	3.39
	Redmon 2003	27	-7.30	6.76	27	-0.8	4.68
	Serrano-Rios 2002	68	-4.5	4.12	65	-1.7	4.03
	Tankova 2003	53	-6.5	5.31	42	-2.7	4.73
Orlistat	Bloch 2003	38	-2.3	2.8	38	-1.5	2.4
	Deeroch 2001	126	-2.6	2.47	126	-1.4	2.47
	Guy-Grand 2001	97	-3.9	3.4	96	-1.3	2.6
	Hanefeld 2002	189	-5.3	5.1	180	-3.4	5.3
	Hawkins 2000	119	-5.4	4.04	118	-2.7	4.02
	Hollander 1998	139	-6.19	6.01	115	-4.31	6.11
	Kelley 2002	137	-3.89	3.16	128	-1.27	3.17
	Kelley 2004	17	-10.1	5.77	22	-9.4	6.1
	Miles 2002	160	-4.7	3.79	139	-1.8	3.54
	Wang 2003	30	-7.0	6.36	31	-3.0	6.36

†Standard deviation

(Data extracted from Norris SL et al. Pharmacotherapy for weight loss in adults with type 2 diabetes mellitus. *Cochrane Database of Systematic Reviews 2005, Issue 1.*)

```
metan n_treat mean_treat sd_treat n_plc mean_plc sd_plc, by(drug) random
second(fixed) lcols (studyname) nostandard xlabel(-10,-8,-6,-4,-2,0,2)
force boxsca(150) texts(100)
```



# Three Measures of Statistical Heterogeneity

- ◆  $Q$
- ◆  $I^2$  statistic
- ◆  $\tau^2$



# Computing Q

$$Q = \sum_{i=1}^k W_i (Y_i - M^*)^2 = \sum_{i=1}^k \left( \frac{Y_i - M^*}{S_i} \right)^2 = \sum_{i=1}^k W_i Y_i^2 - \frac{(\sum_{i=1}^k W_i Y_i)^2}{\sum_{i=1}^k W_i}$$

- ◆ Q is a weighted sum of squares
- ◆ Q is a standardized measure (therefore not affected by the scale used)
- ◆ Under null hypothesis (i.e., all studies share a common effect size), Q follows a chi-squared distribution with  $df=k-1$  (k is # of studies included in the meta-analysis).

## The I<sup>2</sup> Statistic

*What proportion of the observed variance reflects real differences in effect size?*

$$I^2 = \left( \frac{Q - df}{Q} \right) \times 100\%$$

If  $Q < df$ ,  $I^2 = 0$

$$I^2 = \left( \frac{\text{Variance}_{bet}}{\text{Variance}_{total}} \right) \times 100\%$$

- Can be viewed as (not exact)
- Allows us to discuss the amount of variance on a relative scale
- $I^2$  of 25%, 50%, and 75% considered as low, moderate, and high heterogeneity

# Tau-Squared Reflects the Actual Amount of Variation

$\tau^2$  is the between studies variance, its estimate is denoted as  $T^2$

## DerSimonian and Laird Method (method of moment)

$$T^2 = \frac{Q - df}{C}$$

$$Q = \sum_{i=1}^k W_i (Y_i - M^*)^2 = \sum_{i=1}^k W_i Y_i^2 - \frac{(\sum_{i=1}^k W_i Y_i)^2}{\sum_{i=1}^k W_i}$$

$$df = k - 1$$

$$C = \sum W_i - \frac{\sum W_i^2}{\sum W_i}$$

**A caveat:** if the number of studies is small, then the estimate of the  $\tau^2$  will have poor precision

# Measures of Heterogeneity

```
. metan n_treat mean_treat sd_treat n_plc mean_plc sd_plc, random
```

Study	I	SMD	[95% Conf. Interval]	% Weight
1		-0.629	-1.144 -0.114	4.35
2		-0.789	-1.025 -0.553	6.88
3		-0.117	-0.751 0.516	3.51
4		-0.828	-1.079 -0.577	6.74
5		-0.310	-0.559 -0.062	6.76
6		-0.670	-0.932 -0.408	6.64
7		-0.365	-0.571 -0.160	7.14
8		-0.858	-1.153 -0.563	6.32
9		-0.486	-0.736 -0.235	6.74
10		-0.307	-0.759 0.146	4.86
11		-0.751	-1.170 -0.332	5.16
12		-0.687	-1.037 -0.337	5.80
13		-1.118	-1.693 -0.543	3.90
14		-1.528	-1.987 -1.070	4.81
15		-0.451	-0.725 -0.177	6.52
16		-1.856	-2.498 -1.213	3.45
17		-0.357	-0.716 0.003	5.71
18		-1.205	-1.673 -0.736	4.72
D+L pooled SMD		-0.701	<b>-0.859 -0.544</b>	100.00

- CI from a RE meta-analysis describes uncertainty in the location of the mean of systematically different effects in the different studies.
- It does not describe the degree of heterogeneity among studies!

Heterogeneity chi-squared = 66.49 (d.f. = 17) p = 0.000  
 I-squared (variation in SMD attributable to heterogeneity) = 74.4%  
 Estimate of between-study variance Tau-squared = 0.0792

Test of SMD=0 : z= 8.74 p = 0.000

**Q=66.49**  
**I-squared=74.4%**  
**Tau-squared=0.0792**

# Meta-regression and subgroup analysis

# Introduction to Meta-regression

- In primary studies we use regression to assess the relationship between one or more covariates and a dependent variable.
- The same approach can be used with meta-analysis, except that
  - **Unit of analysis** (each observation in the regression model): **individual study** rather than individual participants
  - **Dependent variable: the summary estimate (effect size) in each primary study** rather than outcomes measured in individual participants
  - **Covariates: at level of the study** rather than the level of the participant

# Why do a Meta-regression?

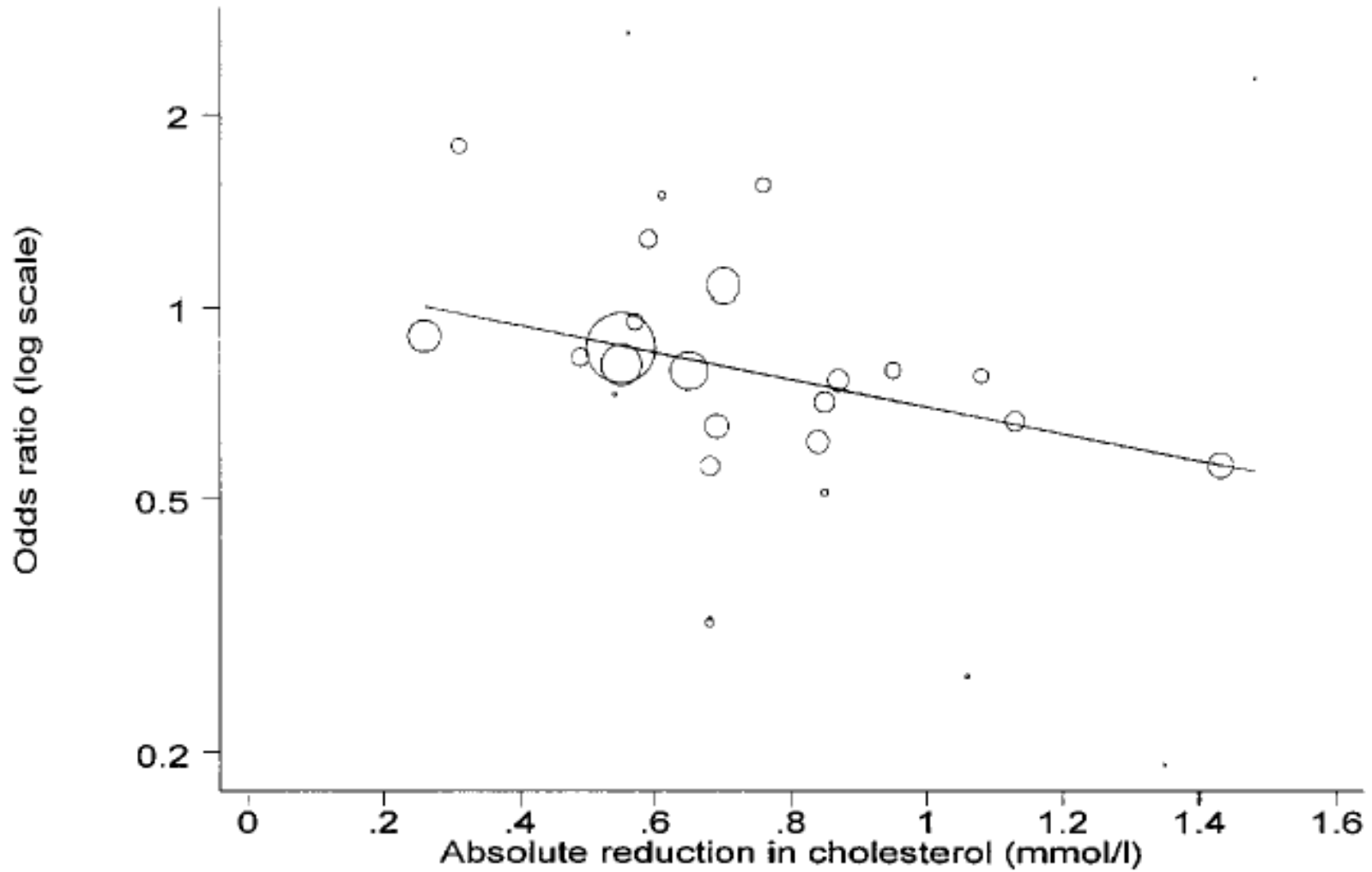
- Examine the relationship between study-level characteristics and effect size

- ***Study potential effect modification:***

- Does the intervention effect (association) vary with different population or study characteristics?*

- Explore and explain between study variation

## Estimated ORs of coronary heart disease in 28 cholesterol reduction RCTs



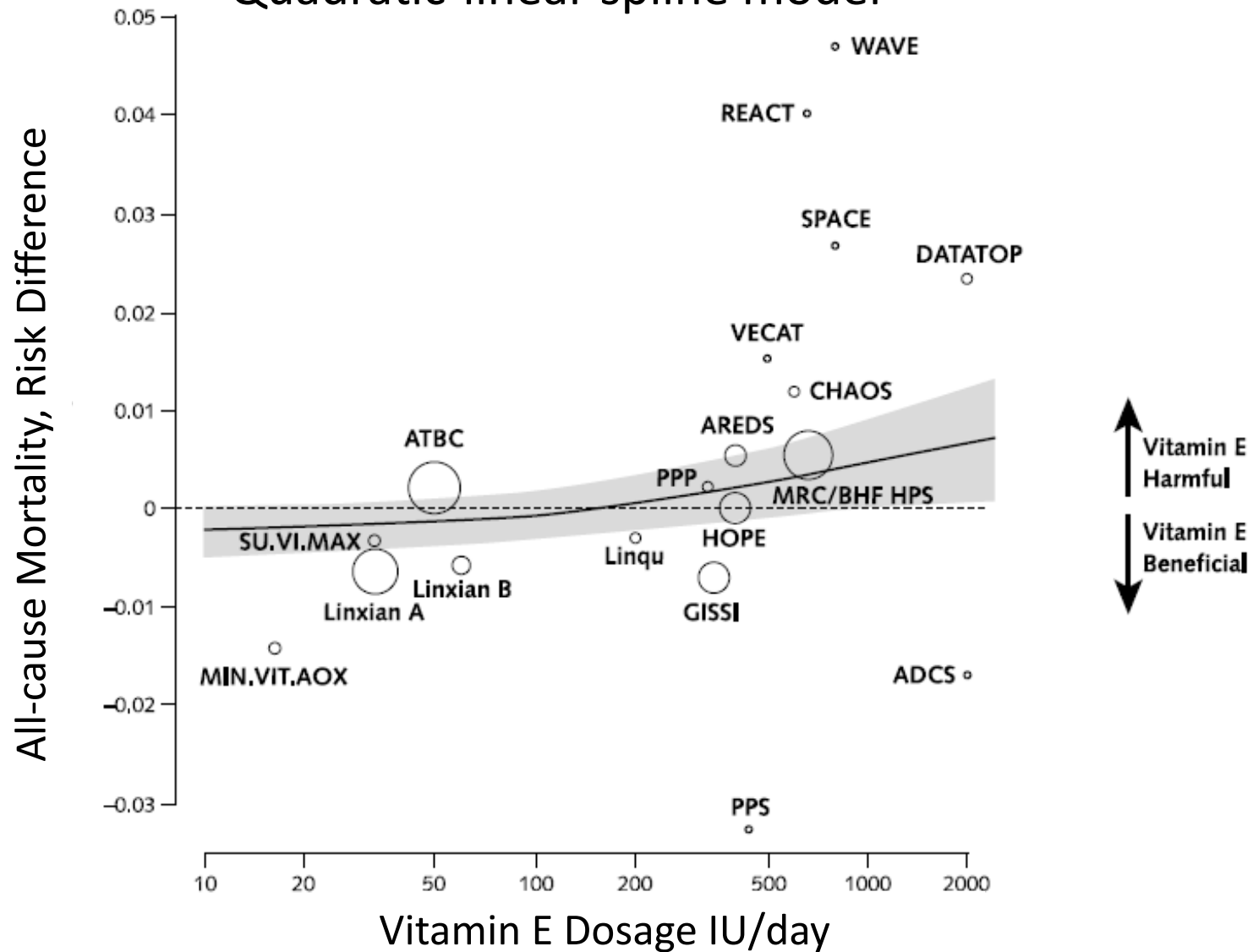


# Introduction to Meta-regression

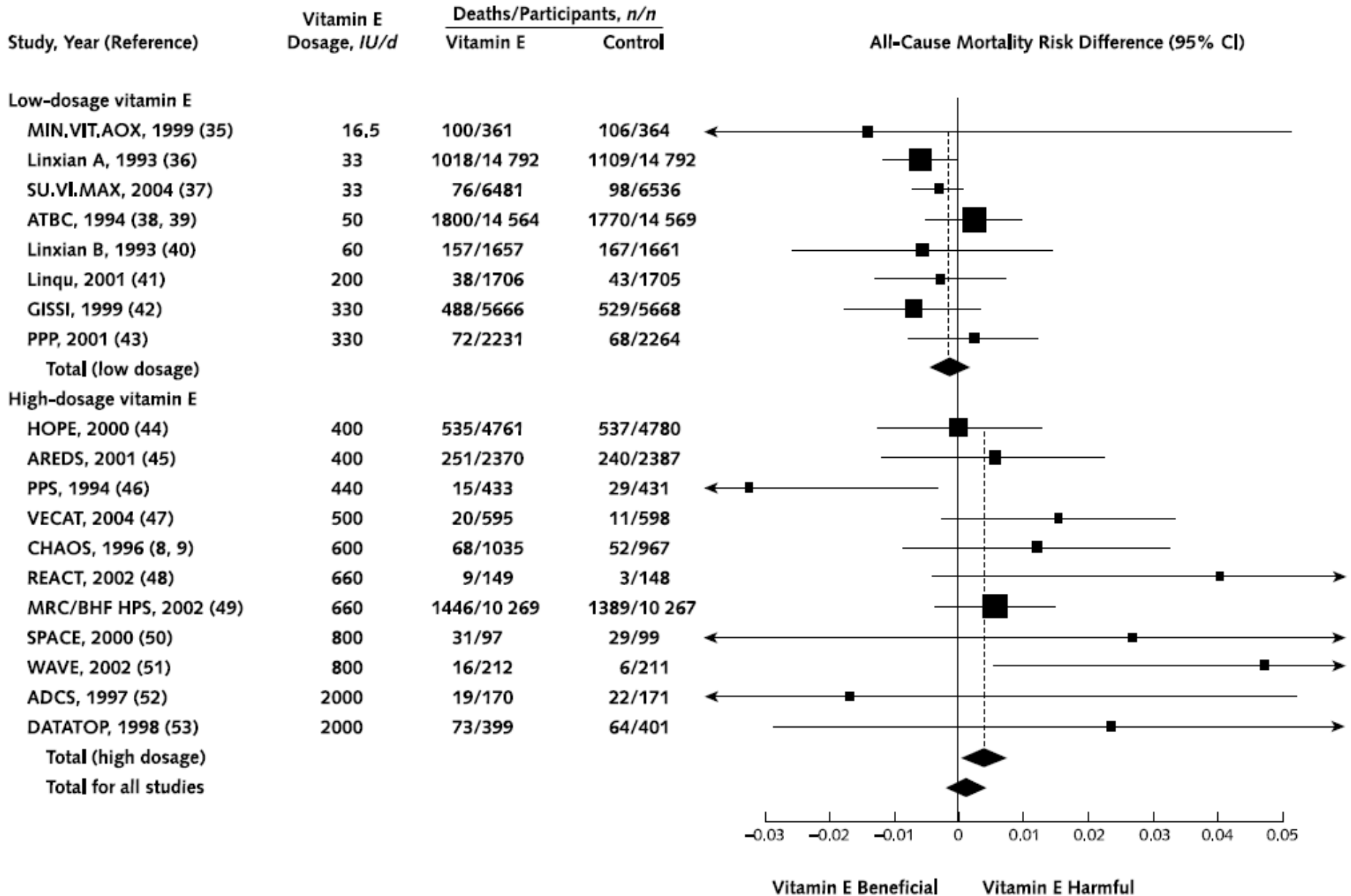
- Coefficient interpretation:  
How outcome variable (the intervention effect) changes on average with a unit change in the explanatory variable (the potential effect modifier).
- Larger studies have more influence on the relationship than smaller studies, since studies are weighted by the precision of their respective effect estimate.
- Both categorical (e.g., dummy-coded) and continuous variables can be used as covariates.
  - **Subgroup analysis:** a special case of meta-regression in which covariates are categorical

# Meta-regression – Vitamin E and All-cause Mortality

Quadratic-linear spline model



# Subgroup Analysis – Vitamin E and All-cause Mortality



# Conducting a Meta-regression

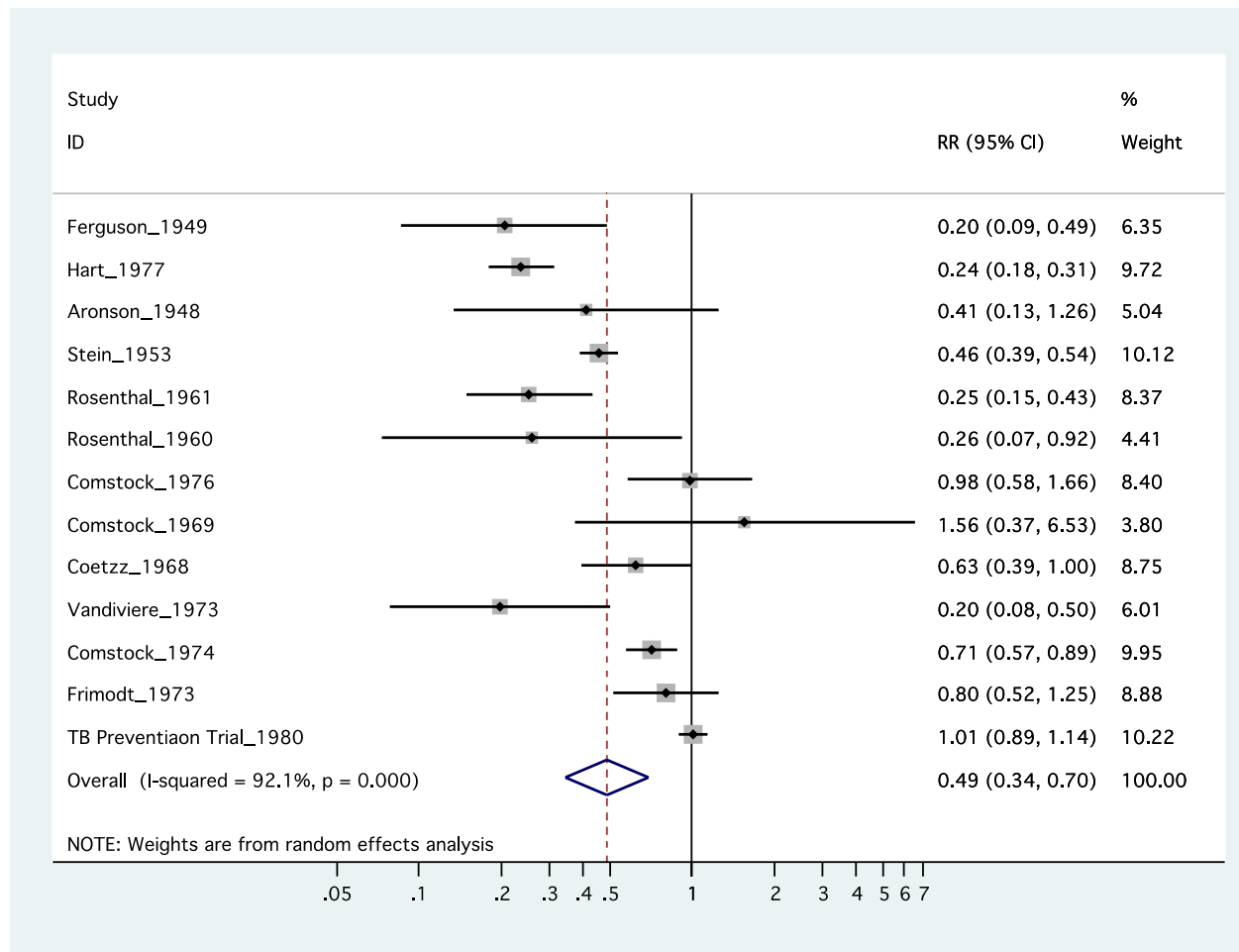
## Bacillus Calmette-Guérin (BCG) Vaccine to Prevent Tuberculosis Dataset

ID	Study	Vaccinated		Control		RR <sup>1</sup>	SE(lnRR)	Latitude <sup>2</sup>
		TB	No TB	TB	No TB			
1	Ferguson_1949	6	300	29	274	0.205	0.441	55
2	Hart_1977	62	13536	248	12619	0.237	0.141	52
3	Aronson_1948	4	119	11	128	0.411	0.571	44
3	Stein_1953	180	1361	372	1079	0.456	0.083	44
4	Rosenthal_1961	17	1699	65	1600	0.254	0.270	42
4	Rosenthal_1960	3	228	11	209	0.260	0.644	42
5	Comstock_1976	27	16886	29	17825	0.983	0.267	33
5	Comstock_1969	5	2493	3	2338	1.562	0.730	33
6	Coetzz_1968	29	7470	45	7232	0.625	0.238	27
7	Vandiviere_1973	8	2537	10	619	0.198	0.472	19
8	Comstock_1974	186	50448	141	27197	0.712	0.111	18
9	Frimodt_1973	33	5036	47	5761	0.804	0.226	13
9	TB Preventiaon Trial 1980	505	87886	499	87892	1.012	0.063	13

1. RR <1.0 indicates the vaccine decreased the risk of TB.
2. The higher the latitude the farther away the study location was from the equator (used as surrogate for climates).

# Heterogeneous Treatment Effects across Studies

```
. metan t_tb t_no_tb c_tb c_no_tb,rr randomi label(namevar=author)
```



$X_H^2$  = chi-squared for heterogeneity = 152.23 (d.f.=12) p=0.000  
 $I^2$  (variation in RR attributable to heterogeneity)=92.1%  
 Estimate of between-study variance Tau-squared =0.3088  
 Test of RR=1 : z=4.00 p =0.000

# Meta-regression Model Specification

$$\ln(RR)_i = a + b * latitude_i + \mu_i + \varepsilon_i$$

$$\varepsilon_i \sim \mathcal{N}(0, (se(\ln RR)_i)^2)$$

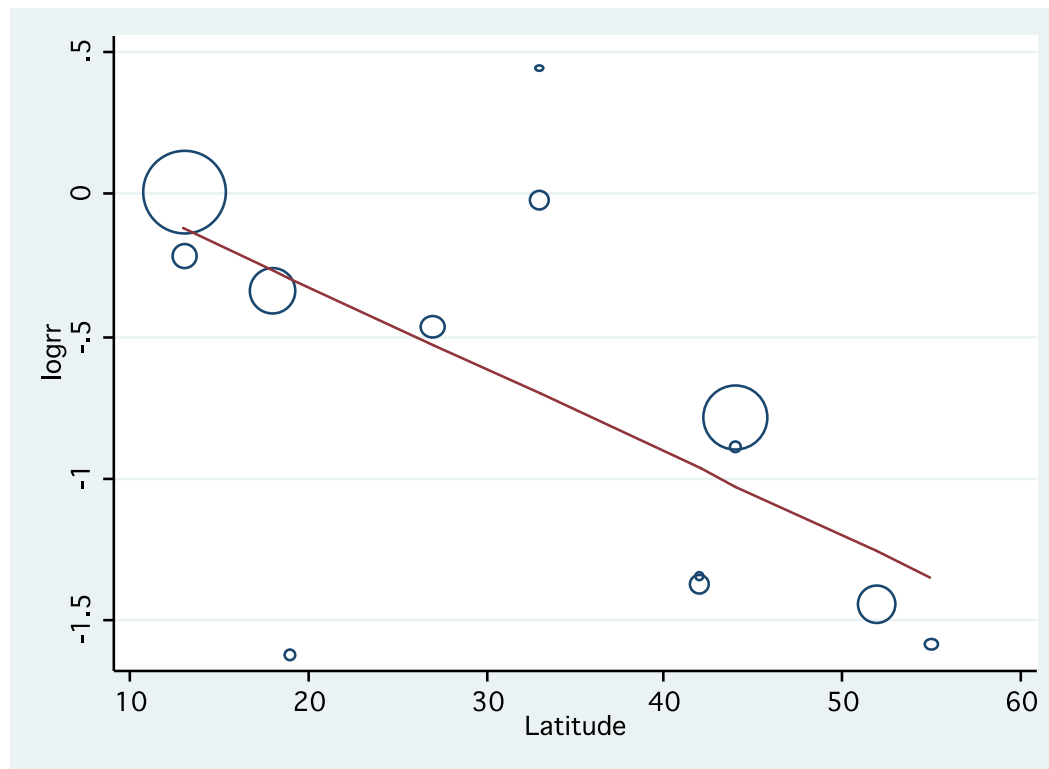
$$\mu_i \sim \mathcal{N}(0, \tau^2)$$

- Parameters to estimate:

- a – intercept,  $\ln(RR)$  at latitude=0 (equator)

- b – slope, the average change in  $\ln(RR)$  for every unit change in latitude

- $\tau^2$  – between study variance



# Interpreting the Coefficients from Meta-regression

```
. metareg logrr latitude, wsse(_selogES) mm graph
```

```
Meta-regression                               Number of obs =      13
Method of moments estimate of between-study variance tau2           =    .0633
% residual variation due to heterogeneity       I-squared_res      =  64.21%
Proportion of between-study variance explained  Adj R-squared      =  79.50%
With Knapp-Hartung modification
```

logrr	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
latitude	-.0292287	.0079378	-3.68	0.004	-.0466996	-.0117579
_cons	.2595437	.2738745	0.95	0.364	-.34325	.8623374

$\ln(\text{RR}) = 0.2595437 - 0.0292287 * \text{latitude}$

**Slope:**

For each unit increase in latitude (moving farther away from equator), the  $\ln(\text{RR})$  measuring the BCG vaccine effectiveness decreased by 0.029 on average. The 95% CI for this estimate is (-0.047 to -0.012) and is statistically significant.

Or on a RR scale,  $\exp(-0.0292287) = 0.97$

–Ratio of two RRs of outcome that are one unit apart in covariate of interest.

– $(\text{RR of TB at latitude } b+1 \text{ unit}) / (\text{RR of TB at latitude } b \text{ unit}) = 0.97$

**Constant on a RR scale:**  $\exp(0.2595437) = 1.30$

At latitude=0 (i.e., equator), the estimated RR of TB was 1.30 (95% CI:0.71 to 2.37) on average comparing those receiving BCG vaccine vs. not receiving vaccine.



# Could Latitude Explain Some of the Variation?

- Intercept only model

```
. metareg logrr, wsse(_selogES) mm
```

Meta-regression  
 Method of moments estimate of between-study variance  
 % residual variation due to heterogeneity  
 With Knapp-Hartung modification

Number of obs	=	13
tau2	=	.3088
I-squared_res	=	92.12%

logrr	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
_cons	-.7141172	.1806966	-3.95	0.002	-1.107821	-.3204131

- With latitude in the model

```
. metareg logrr latitude, wsse(_selogES) mm graph
```

Meta-regression  
 Method of moments estimate of between-study variance  
 % residual variation due to heterogeneity  
 Proportion of between-study variance explained  
 With Knapp-Hartung modification

Number of obs	=	13
tau2	=	.0633
I-squared_res	=	64.21%
Adj R-squared	=	79.50%

logrr	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
latitude	-.0292287	.0079378	-3.68	0.004	-.0466996	-.0117579
_cons	.2595437	.2738745	0.95	0.364	-.34325	.8623374

- Proportion of total between-study variance explained by the model:

$$R^2 = \frac{\tau_{\text{explained}}^2}{\tau_{\text{total}}^2} = \frac{\tau_{\text{total}}^2 - \tau_{\text{unexplained}}^2}{\tau_{\text{total}}^2} = 1 - \frac{\tau_{\text{unexplained}}^2}{\tau_{\text{total}}^2} = 1 - \left( \frac{0.0633}{0.3088} \right) = 0.7950$$

# Variance(Heterogeneity) Explained by a Covariate

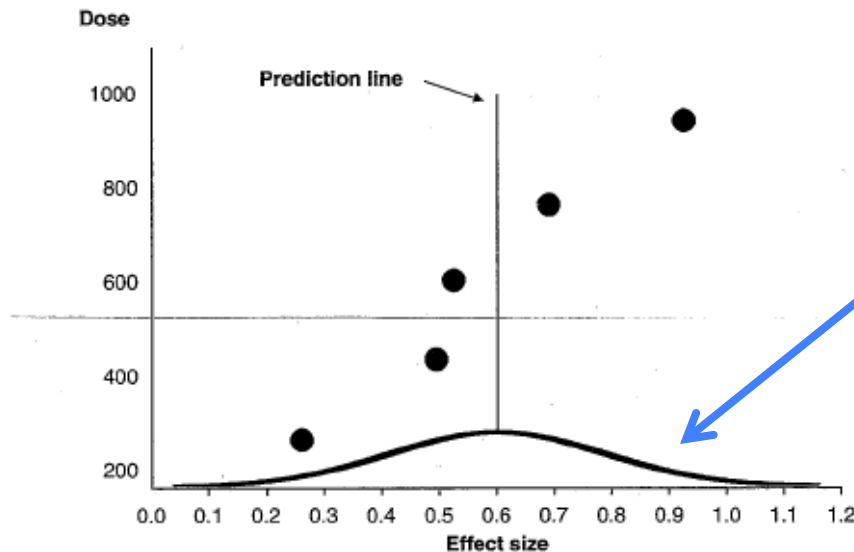


Figure 20.7 Between-studies variance ( $\tau^2$ ) with no covariate.

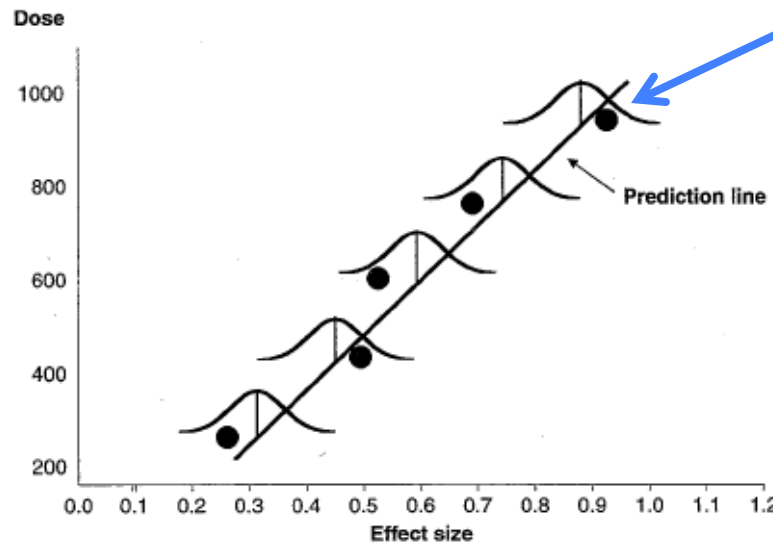


Figure 20.8 Between-studies variance ( $\tau^2$ ) with covariate.

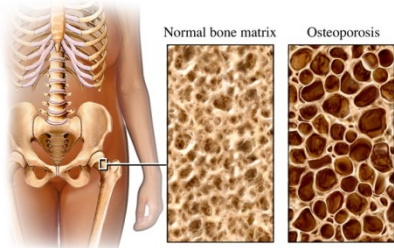
The spread of this distribution reflects the amount of between study variance ( $\tau^2$ ) without any covariate.

The spread of this distribution reflects the amount of between study variance with a covariate; assumed to be the same at each level of covariate.

The decrease in spread from the top to the bottom pane illustrates how a covariate explains some of the between-studies variance.

# Introduction to Indirect Comparison

## Which Treatment Should be Recommended?



The NEW ENGLAND JOURNAL of MEDICINE

CLINICAL THERAPEUTICS



A 67-year-old woman was referred by her primary care physician for treatment of osteoporosis and progressive bone loss. One year before the visit, the patient had discontinued hormone-replacement therapy. She had subsequently begun to experience midback pain and lost 1.5 inch in height. A x-ray scan has confirmed a diagnosis of osteoporosis. One year later, a second scan showed a further decrease of bone mineral density at the lumbar spine, as well as a compression fracture of the 11th thoracic vertebra.

**Which treatment should be recommended?**

Paraphrased from  
Favus NEJM 2010

■ **Medical treatment:**

**Over 10 drugs/combination of drugs**

- ✓ Estrogen
- ✓ Selective estrogen receptor modulators (SERMs)- Raloxifene
- ✓ Calcium and/or vitamin D
- ✓ Bisphosphonates, e.g., alendronate (Fosamax), risedronate (Actonel)
- ✓ Other hormones, e.g., Teriparatide (Forteo)

■ **Cost: ranges from \$4 to \$130 per month**

**Where is the evidence?**

## 14 Cochrane systematic reviews

### **Which interventions work? In Whom?**

*“At a dose of 10 mg per day, **alendronate** results in a **statistically significant and clinically important reduction** in vertebral, non-vertebral, hip and wrist fractures (Wells 2010).”*

*“**No statistically significant reductions** in non-vertebral, hip, or wrist fractures were found, regardless of whether **etidronate** was used for primary or secondary prevention (Wells 2010).”*

*“**Vitamin D alone** appears **unlikely to be effective in preventing hip fracture...Vitamin D with calcium** reduces hip fractures (Avenell 2009).”*

# Indirect comparison

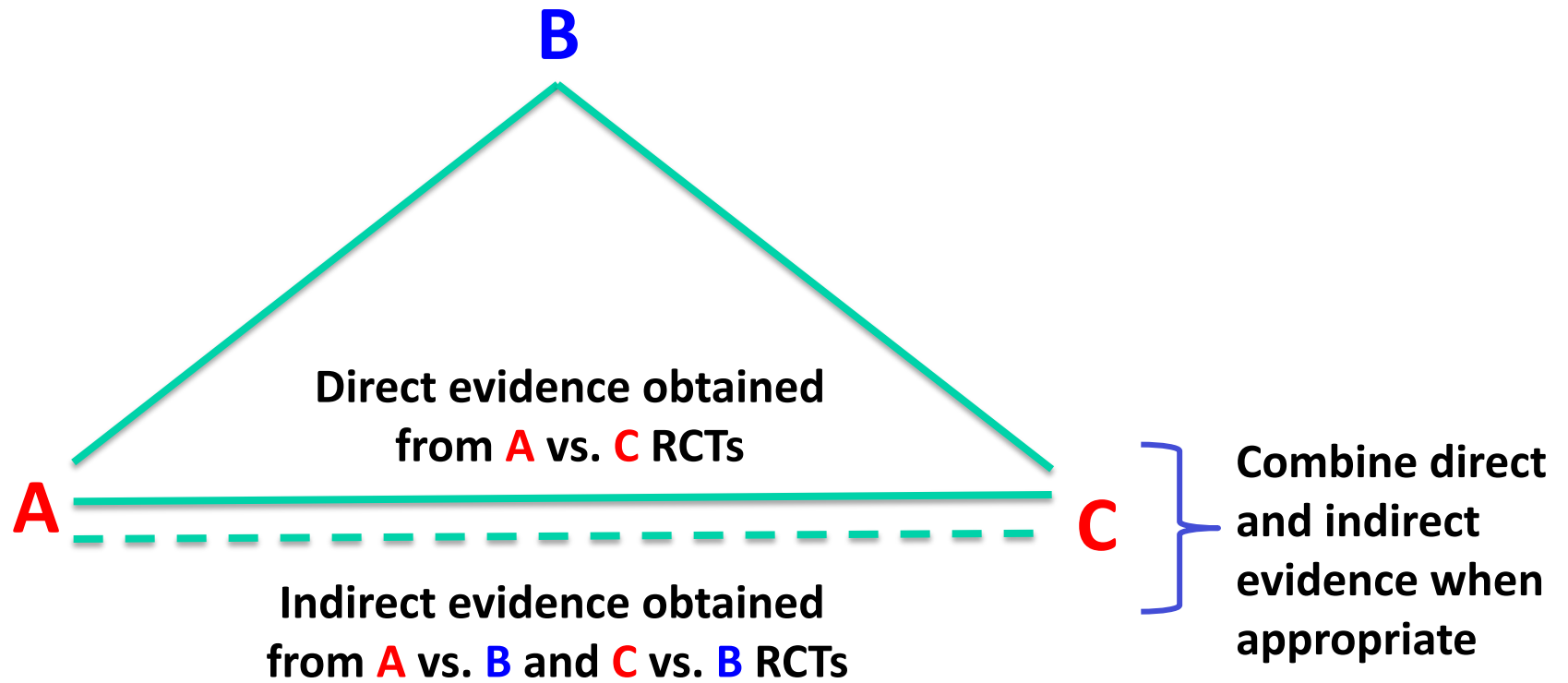
- If we know how much taller is B to A and how much taller is C to A we know how much taller is B compared to C



- For any pair B and C,

Typical (or mean) advantage of C over B =  
advantage of C over A – advantage of B over A

# Indirect Comparison and Network Meta-analysis Framework

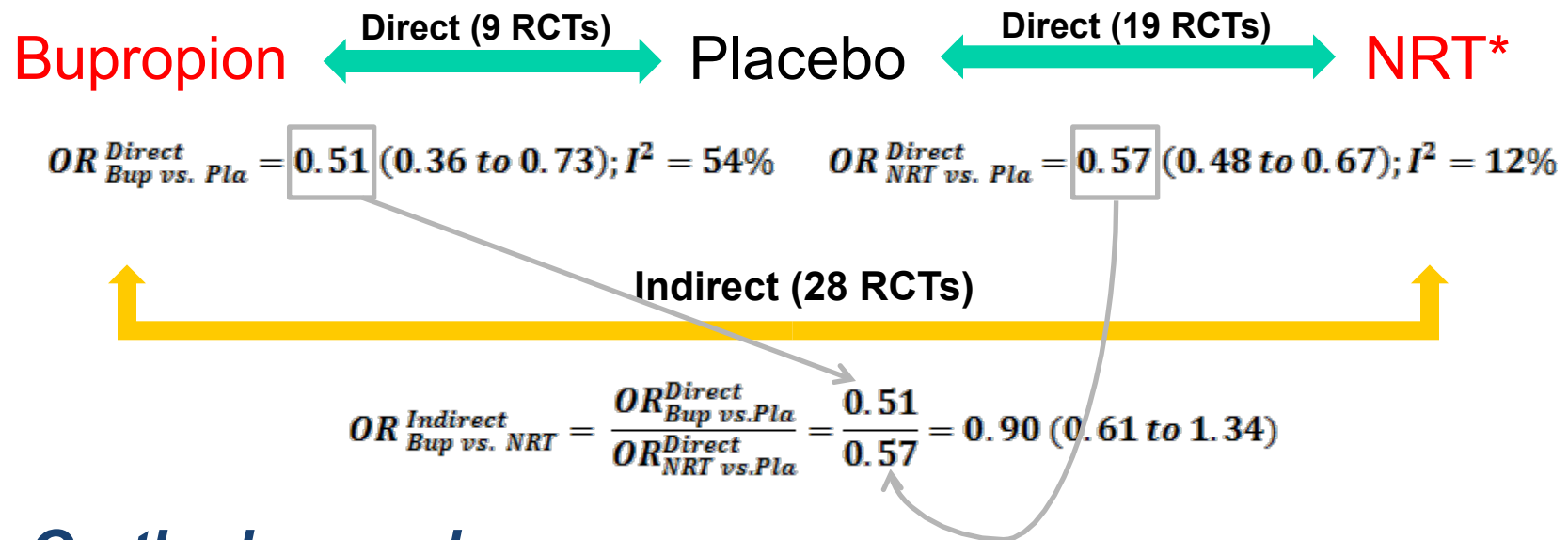


Solid line: direct evidence  
Dashed line: indirect evidence

Bucher 1997; Song 2003; Glenny 2005



## Indirect Comparison Formulation – A Simple Example



**On the log scale:**

$$\log(OR_{Bup\ vs.\ NRT}^{Indirect}) = \log(OR_{Bup\ vs.\ Pla}^{Direct}) - \log(OR_{NRT\ vs.\ Pla}^{Direct})$$

$$\begin{aligned} Var[\log(OR_{Bup\ vs.\ NRT}^{Indirect})] \\ = Var[\log(OR_{Bup\ vs.\ Pla}^{Direct})] + Var[\log(OR_{NRT\ vs.\ Pla}^{Direct})] \end{aligned}$$

\* NRT: Nicotine Replacement Therapy



# Statistical considerations in indirect comparisons and network meta-analysis

Saïd Business School, Oxford, UK

March 18-19, 2013