

# INTRODUCTION TO META-ANALYSIS 3

## *dealing with heterogeneity*

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# General principles of meta-analysis

- Participants of one study are not compared directly with the participants in another study
  - each study is analyzed separately
  - in each study we estimate the intervention effect preserving the randomization (e.g. RR, OR)
- In each study we assign a weight depending on the information it provides
  - in a way that large studies have greater influence in the summary effect
- The study-specific intervention effects are synthesized to obtain the summary effect of the meta-analysis

# Why performing a meta-analysis?

- To increase the power of the analysis and get more precise results
  - obtaining narrower confidence intervals
  - detecting statistically significant effects

# Why performing a meta-analysis?

- To increase the power of the analysis and get more precise results
  - obtaining narrower confidence intervals
  - detecting statistically significant effects
  
- To investigate the intervention effect under different conditions
  - exploration of heterogeneity

# What is heterogeneity?

- The differences observed between the studies of a systematic review.
- Types of heterogeneity – diversity:
  1. Clinical
  2. Methodological
  3. Statistical

# Clinical heterogeneity

- Participants

- Age
- Severity of condition
- Geographical variation

- Interventions

- Intensity / dose / duration
- Sub-type of drug
- Mode of administration,
- Nature of the control (placebo/none/standard care)

# Methodological heterogeneity

- Design
  - Randomised vs non-randomised
  - Cross-over vs parallel group vs cluster randomised
  - Follow-up duration
- Conduct
  - Allocation concealment
  - Blinding
  - Analysis method
- Outcomes
  - Definition of an event
  - Choice of measurement scale



# Statistical heterogeneity

- Effect estimates will vary across studies
- Some variation is chance variation:
  - Studies are small
  - All results come with uncertainty
  - Effect estimates will vary by chance
- Some variation is genuine differences in the effect across studies
  - Clinical / methodological heterogeneity
- Statistical heterogeneity is the observed variation in effect estimates that cannot be explained by chance alone

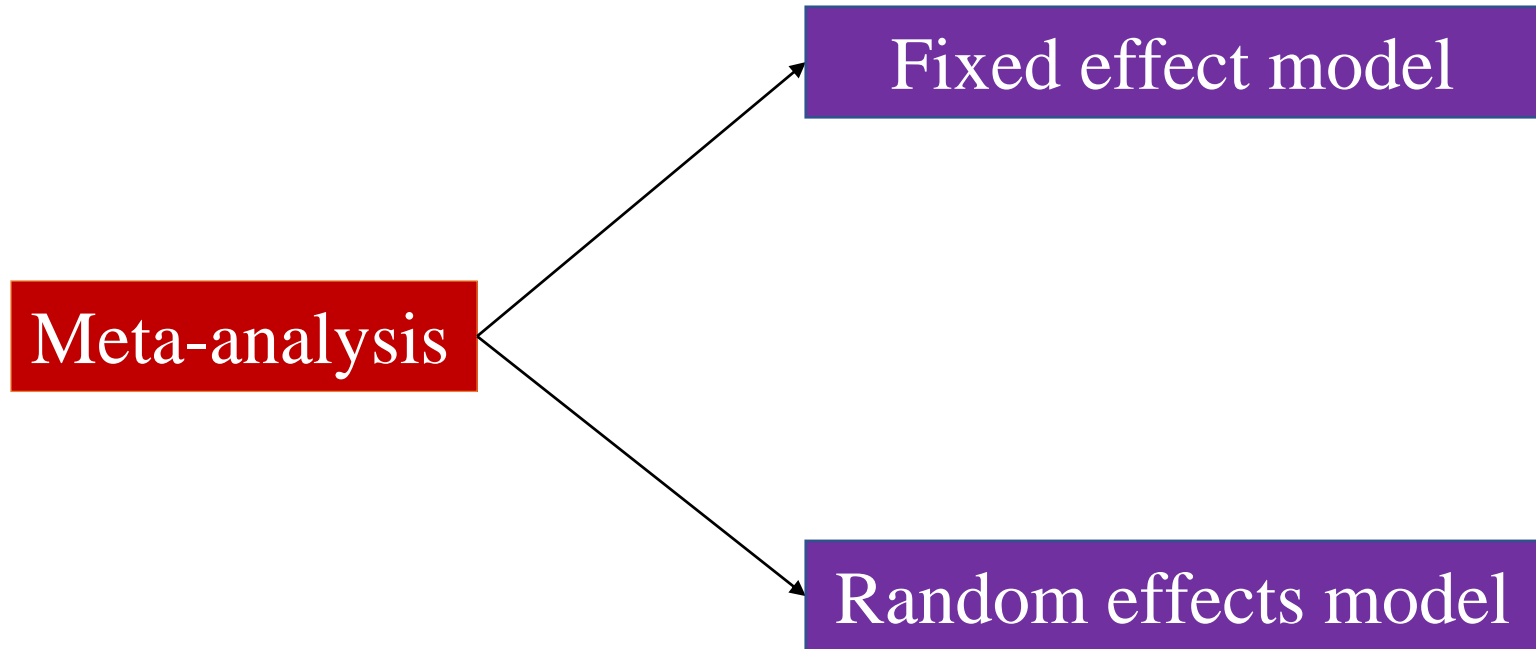
# Outcome data required from each study

- Extract from each study an effect size and its uncertainty (standard error)
- Usually we present the effect sizes from all studies in a forest plot

# How to synthesize these studies?

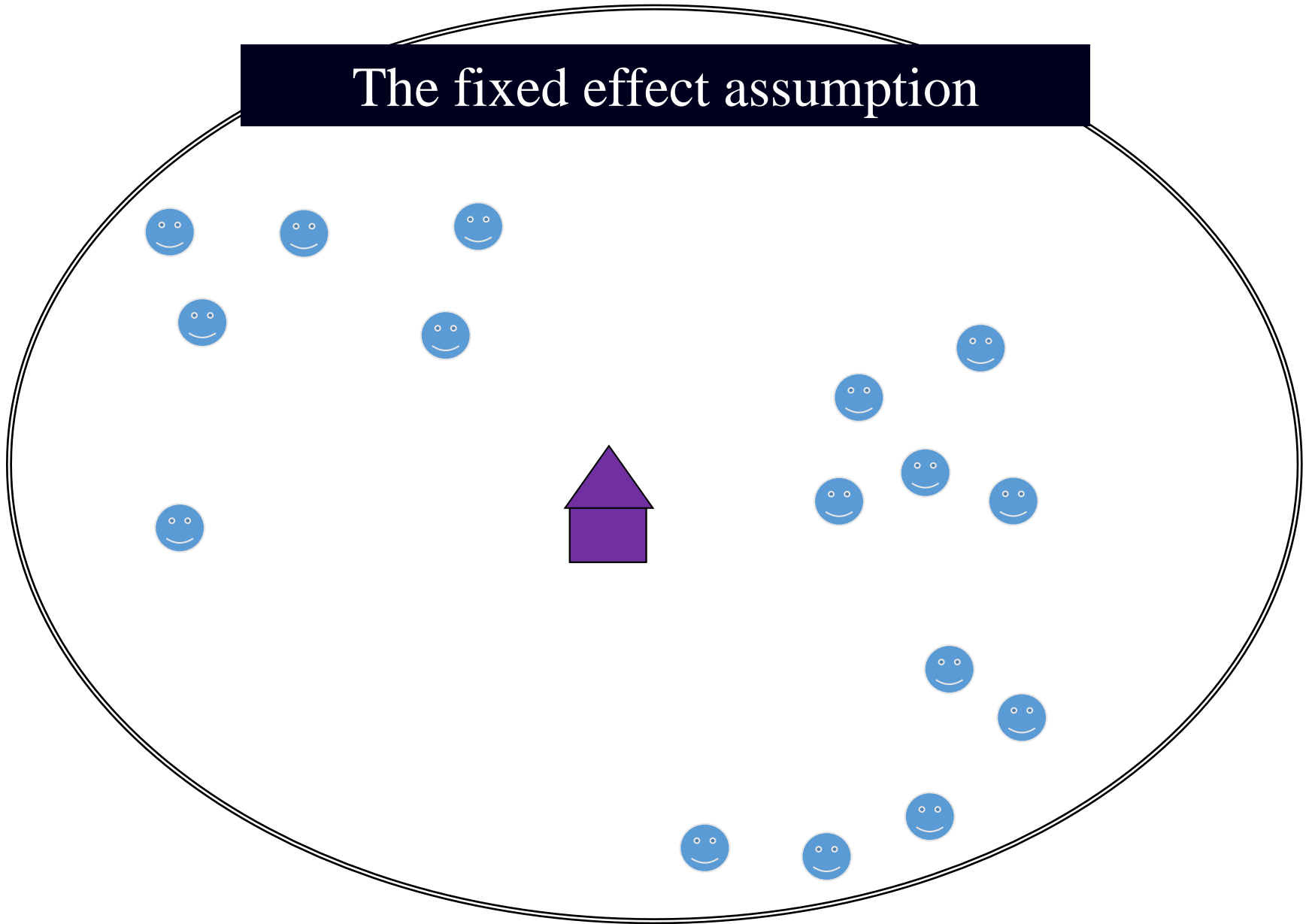
- By obtaining a ~~average~~ effect
  - Differences in level of uncertainty across the studies are ignored
- By pooling the different ~~intervention~~ arms across all studies
  - this approach breaks the randomization of the studies – comparison between treatment and control valid within studies but potentially invalid across studies
- By obtaining a weighted average
  - Randomization is preserved and larger (more precise) studies have larger weight in the analysis

# Meta-analysis models

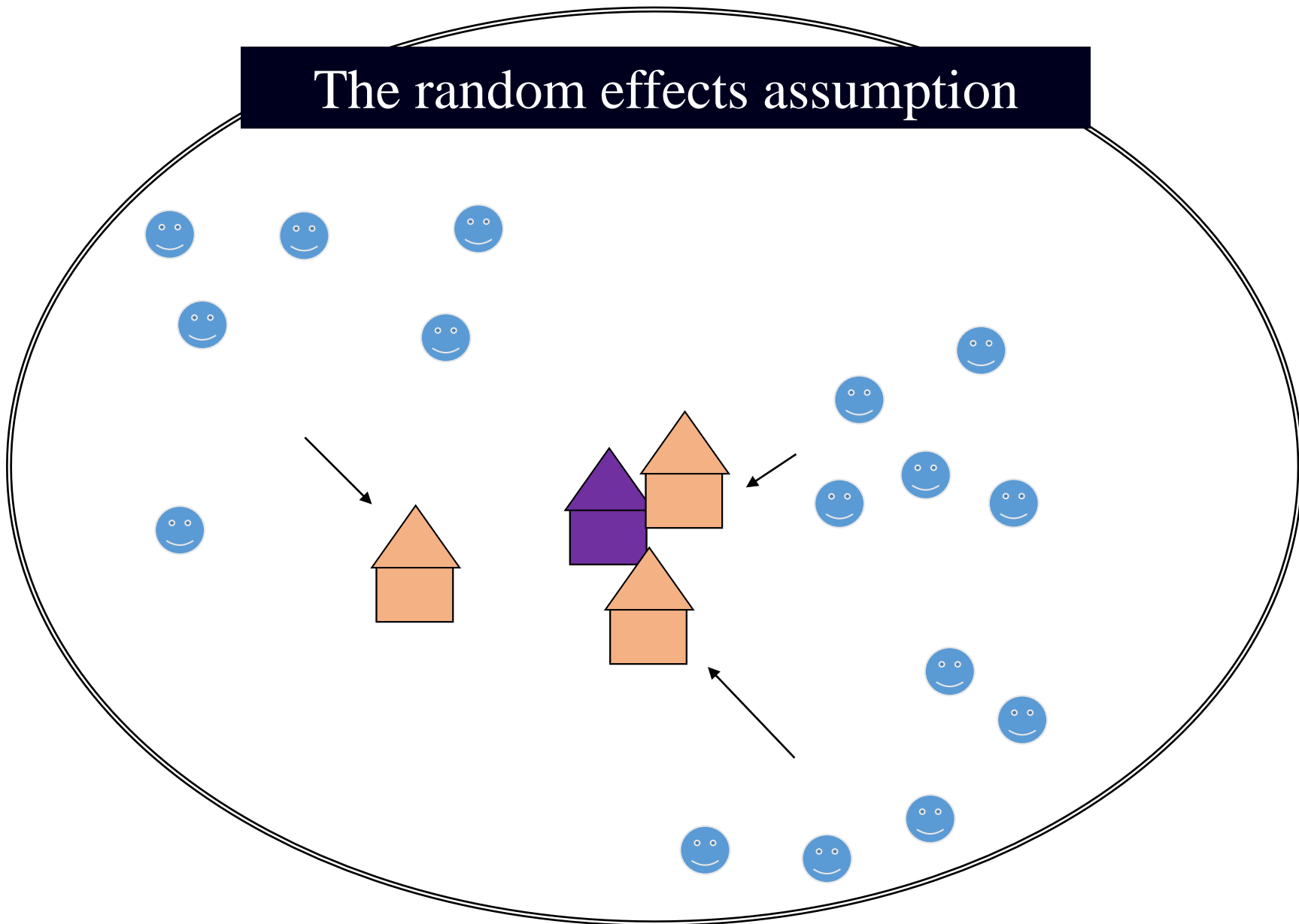


**What are these models?**

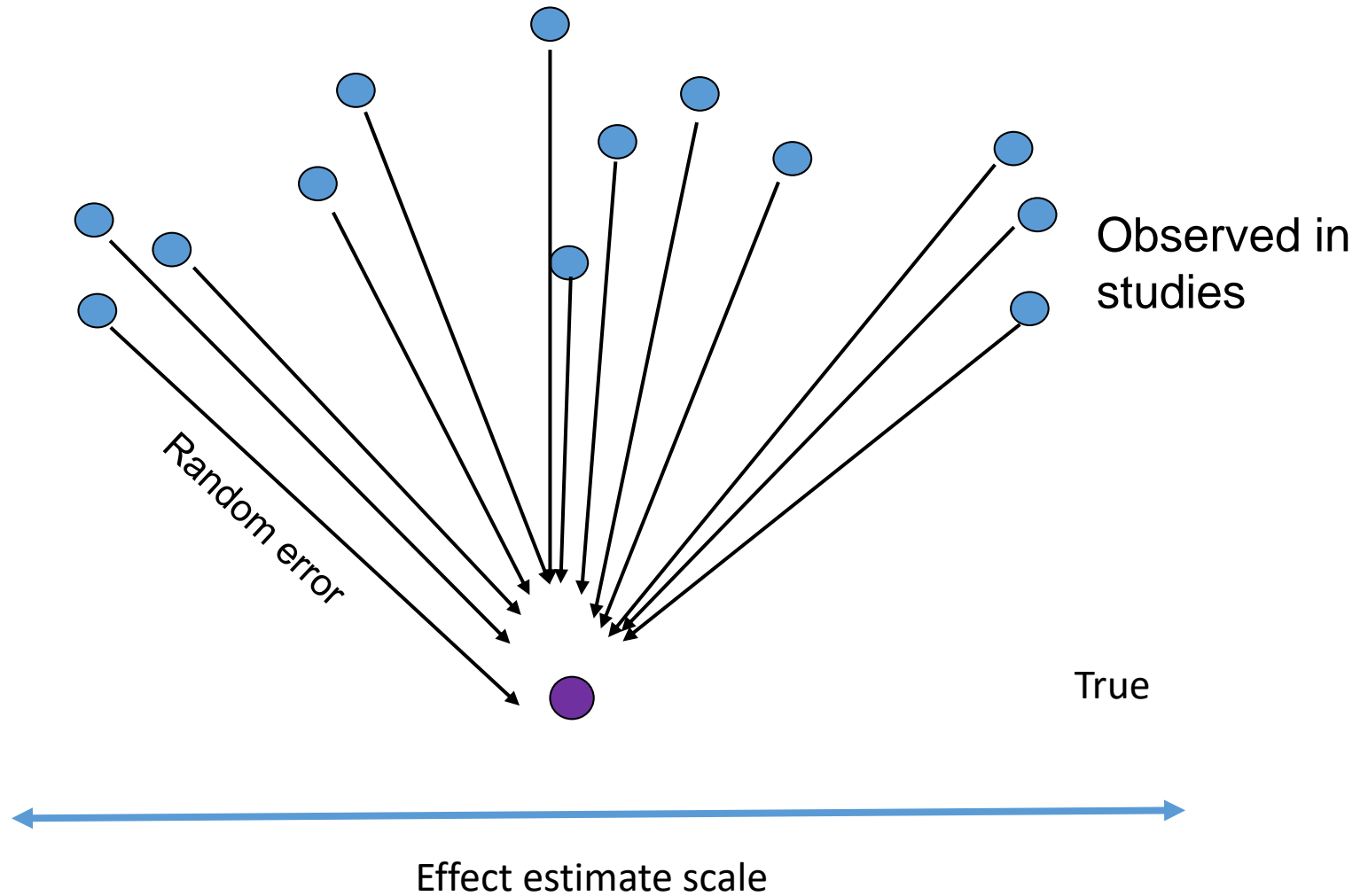
# The fixed effect assumption



# The random effects assumption

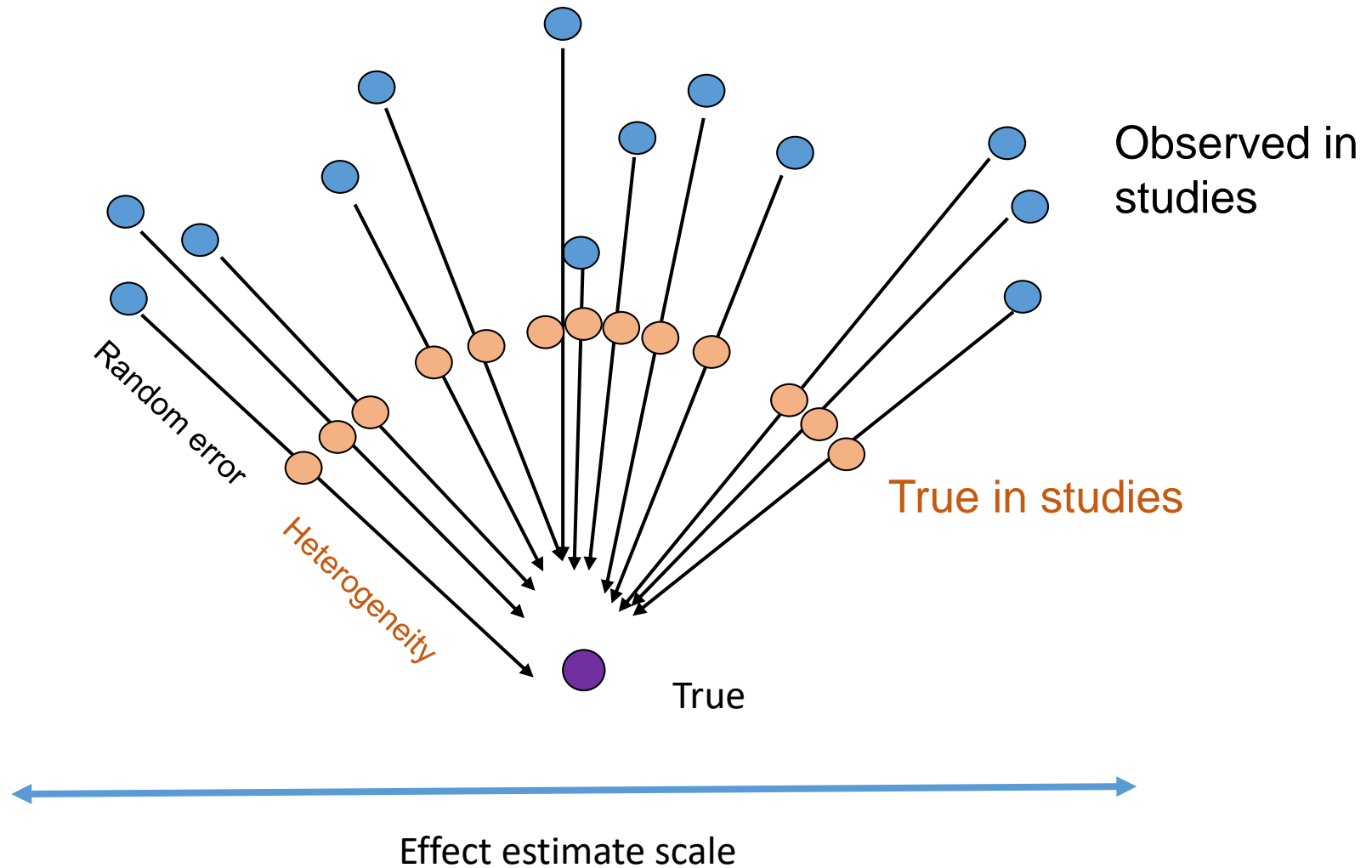


# The fixed effect assumption





# The random effects assumption



# How to assign weights to the studies?

- Inverse variance method
  - any type of data, both fixed and random effects
  - in fact this is the maximum likelihood estimator!
- Mantel-Haenszel method
  - only binary data, only fixed effect (but there are ways to account for the heterogeneity)
- Peto method
  - only binary data, only odds ratio, only fixed effect

# Fixed effect model

- Inverse variance method
- Weight is  $1 \div \text{variance}$

$$w_i = \frac{1}{\text{var}(\hat{y}_i)} \quad \text{for each study } i$$

$$\hat{\theta}_{FE} = \frac{\sum w_i \hat{y}_i}{\sum w_i} \quad \text{var}(\hat{\theta}_{FE}) = \frac{1}{\sum w_i}$$

# Random effects model

- Inverse variance method
- Uncertainty in each trial is now BOTH the random variation AND the heterogeneity
- Weight is  $1 \div (\text{variance} + \text{heterogeneity})$

$$w_i^* = \frac{1}{\text{var}(\hat{y}_i) + \tau^2} \quad \text{for each study } i$$

$$\hat{\theta}_{FE} = \frac{\sum w_i^* \hat{y}_i}{\sum w_i^*} \quad \text{var}(\hat{\theta}_{FE}) = \frac{1}{\sum w_i^*}$$

The weights are smaller than before

# Example: Organized inpatient rehabilitation

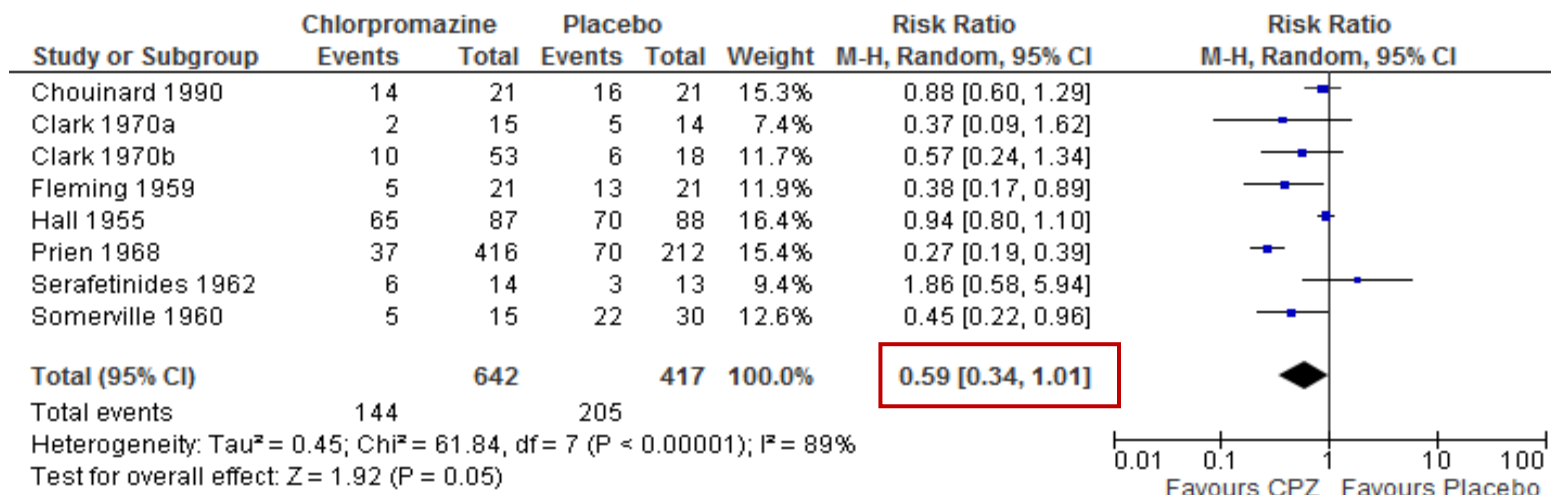
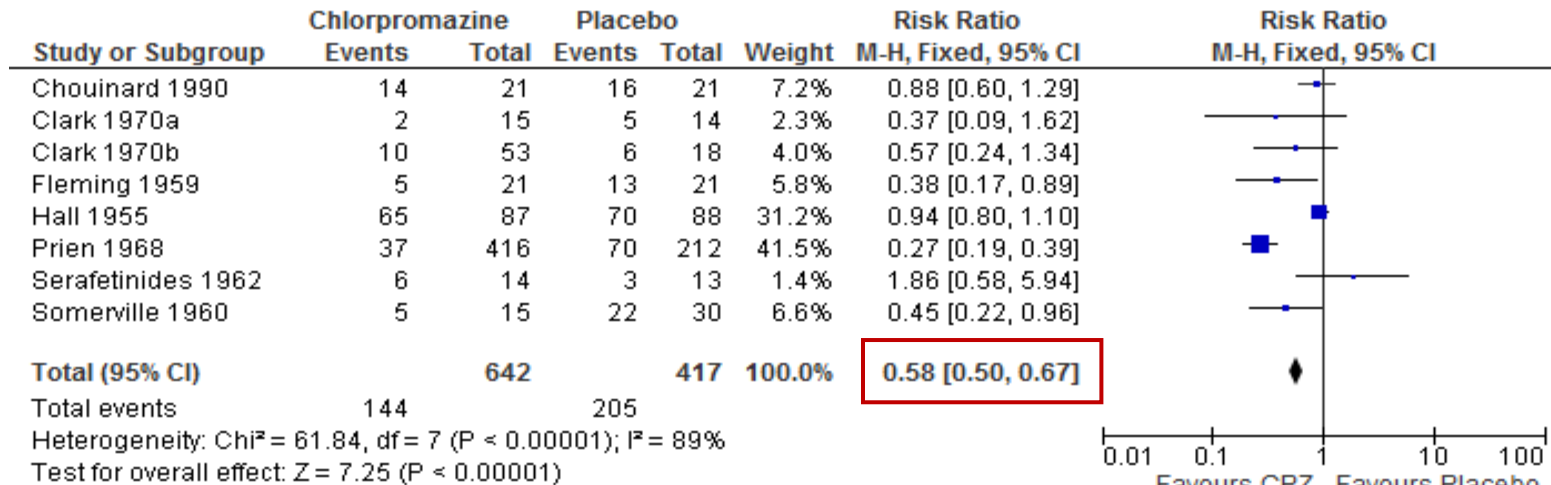
	OR	ln (OR)	var	weight FE		weight RE	
Study		$y_i$	$v_i$	$w_i$	$w_i y_i$	$w_i^*$	$w_i^* y_i$
<b>Cameron 1993</b>	0.98	-0.02	0.10	10.0	-0.2	7.6	-0.2
<b>Fordham 1986</b>	1.36	0.31	0.26	3.8	1.2	3.4	1.1
<b>Galvard 1995</b>	1.28	0.25	0.06	16.6	4.2	10.9	2.7
<b>Gilchrist 1988</b>	0.75	-0.29	0.14	7.1	-2.1	5.8	-1.7
<b>Kennie 1988</b>	0.45	-0.79	0.21	4.8	-3.8	4.1	-3.3
<b>Total</b>				<b>42.3</b>	<b>-0.65</b>	<b>31.8</b>	<b>-1.3</b>

- Random effects meta-analysis
  - pooled odds ratio =  $\exp\{-0.045\} = 0.96$
  - 95% confidence interval from 0.68 to 1.35
- Fixed effect analysis
  - pooled odds ratio =  $\exp\{-0.02\} = 0.98$
  - 95% confidence interval from 0.72 to 1.32

Random effects model gives wider confidence intervals!

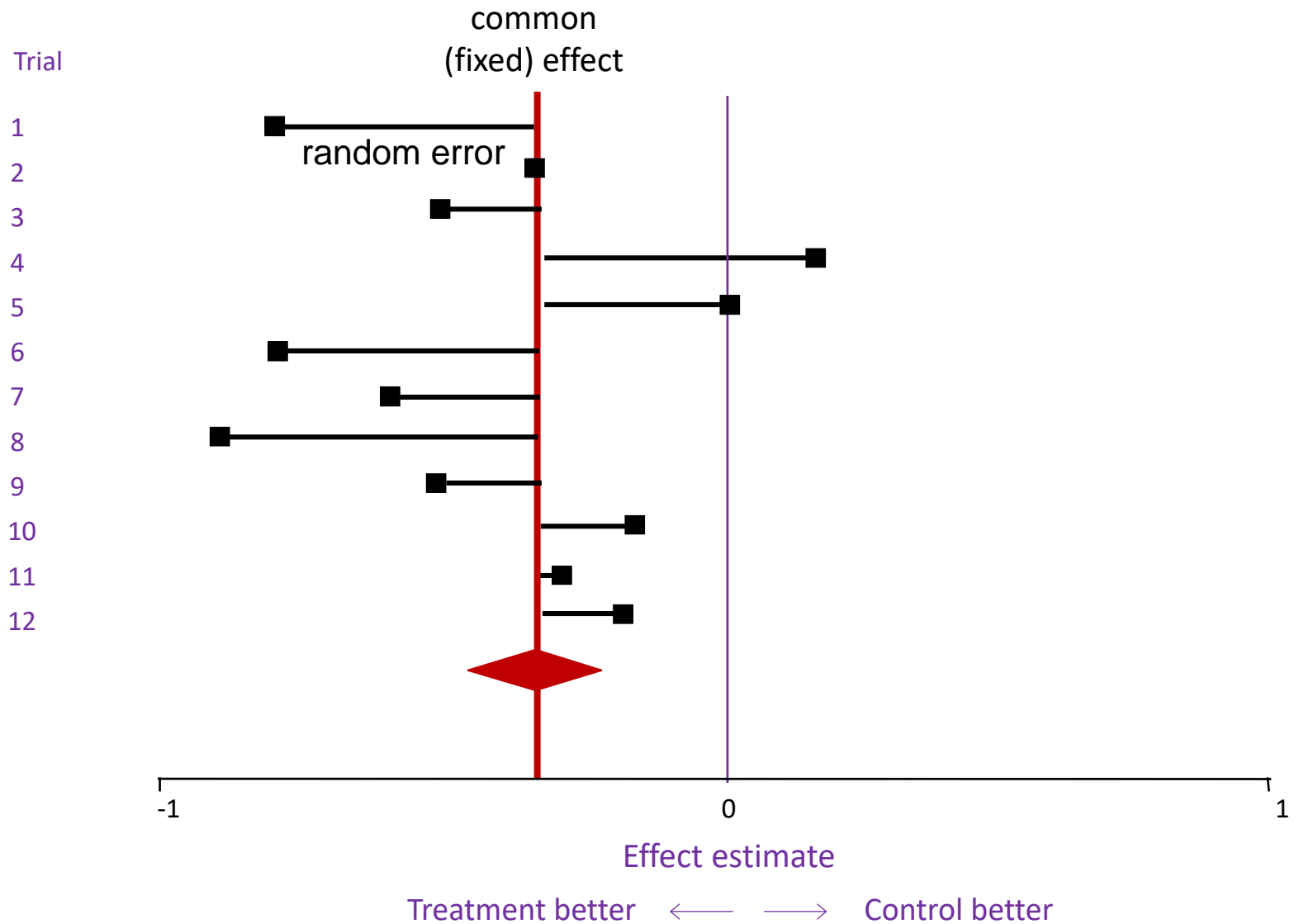
# Example: Behaviour

## Deteriorated/Disturbed/Unco-operative

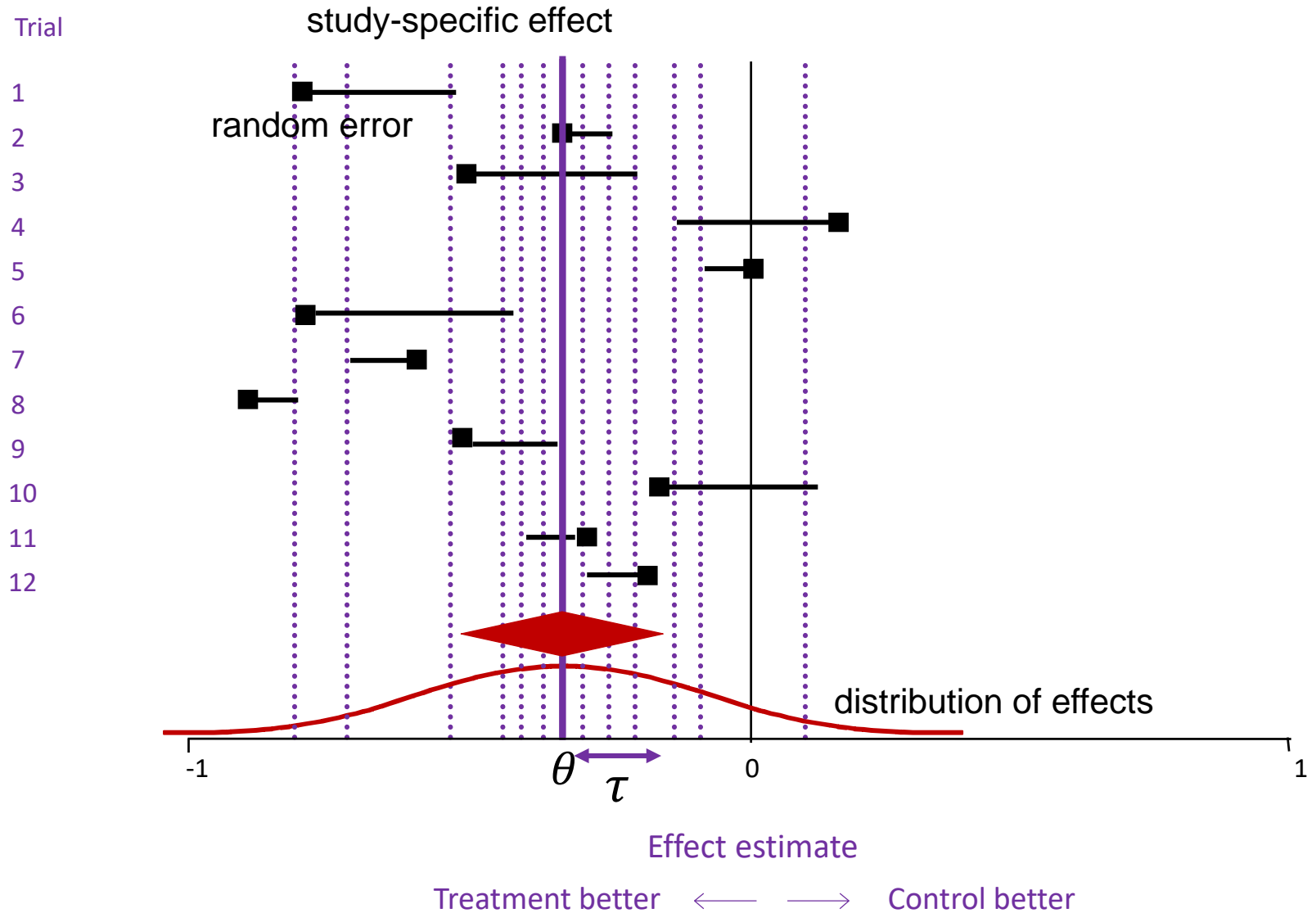


RE gives more conservative results

# Fixed effect meta-analysis



# Random effects meta-analysis





# Random effects meta-analysis

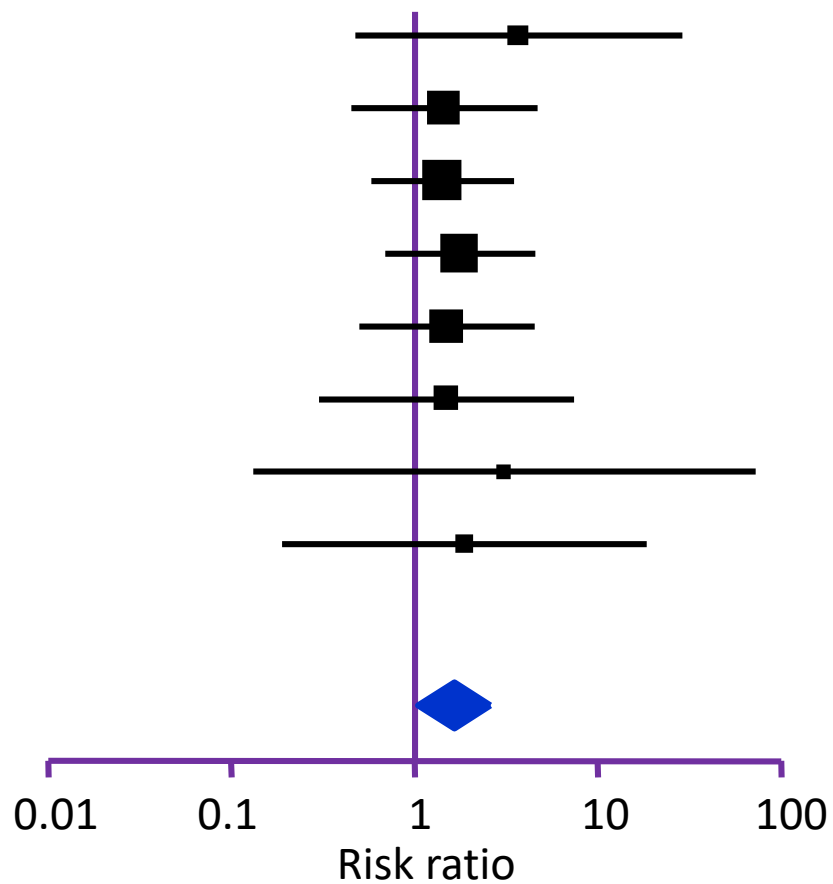
- Heterogeneity suggests that the studies have important underlying differences.
- We can allow the true effects underlying the studies to differ.
- We assume the true effects underlying the studies follow a distribution.
  - conventionally a normal distribution
- We use a simple adaptation of the inverse-variance weighted average.

# Identifying heterogeneity

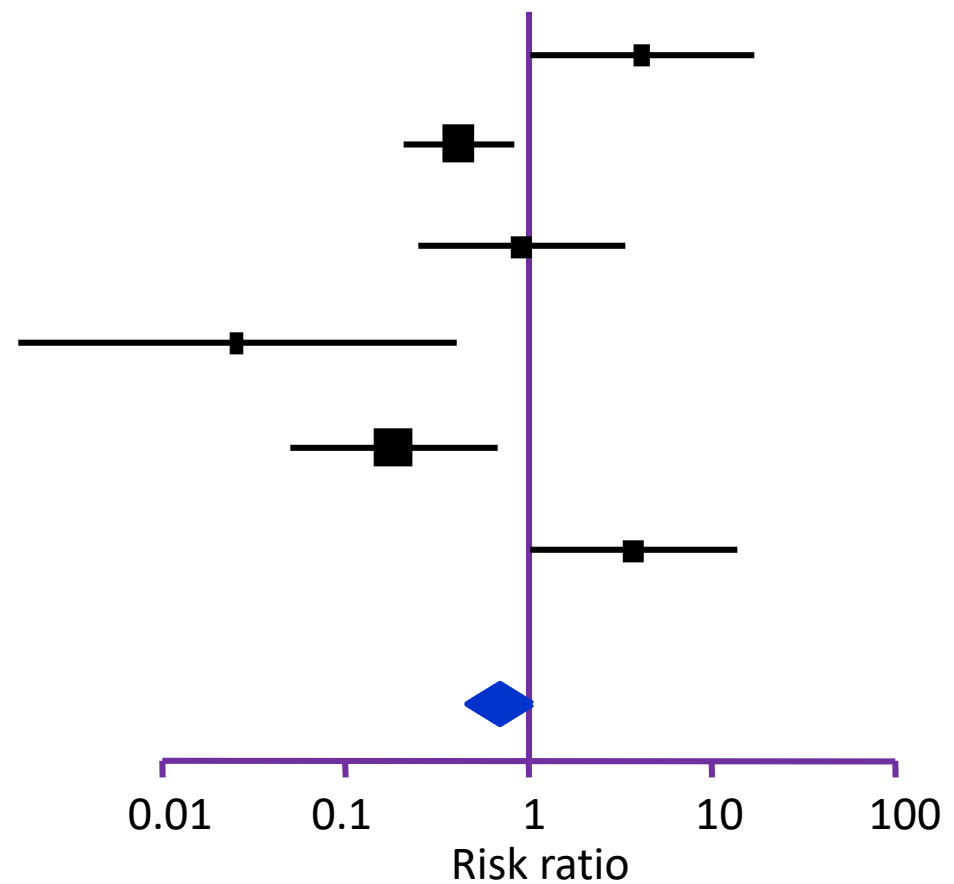
1. **Visual** inspection of the forest plots
2. **Q** test for the presence of heterogeneity
3. **I<sup>2</sup>** statistic that quantifies heterogeneity as a proportion

# Visual inspection of the forest plot

- A graphical inspection of the results is usually the first step
- A lack of overlap in confidence intervals indicates heterogeneity



Favours treatment ← → Favours placebo



Favours treatment ← → Favours control

# Q-test

- chi-squared ( $\chi^2$ ) test

$$Q = \sum w_i (\hat{y}_i - \hat{\theta})^2$$

- has  $\chi^2$  distribution with  $k - 1$  d.f. under null hypothesis of an identical effect in every study
- $k$  is the number of studies in the meta-analysis
- rejection of  $H_0$  suggests heterogeneity

# Q-test drawbacks

- Has low power since there are usually very few studies
  - i.e. test is not very good at detecting heterogeneity as statistically significant when it exists
- But, has excessive power to detect clinically unimportant heterogeneity when there are many studies

# I-square statistic

Higgins and Thompson (2002)

- Q-test is not asking a useful question if heterogeneity is inevitable
- Quantify heterogeneity
  - based on the  $\chi^2$  statistic  $Q$  and its degrees of freedom

$$I^2 = \frac{\textit{Heterogeneity}}{\textit{Heterogeneity} + \textit{Average study variance}}$$

$$I^2 = \frac{Q - k + 1}{Q} * 100\%$$

describes the proportion of total variability that is due to heterogeneity

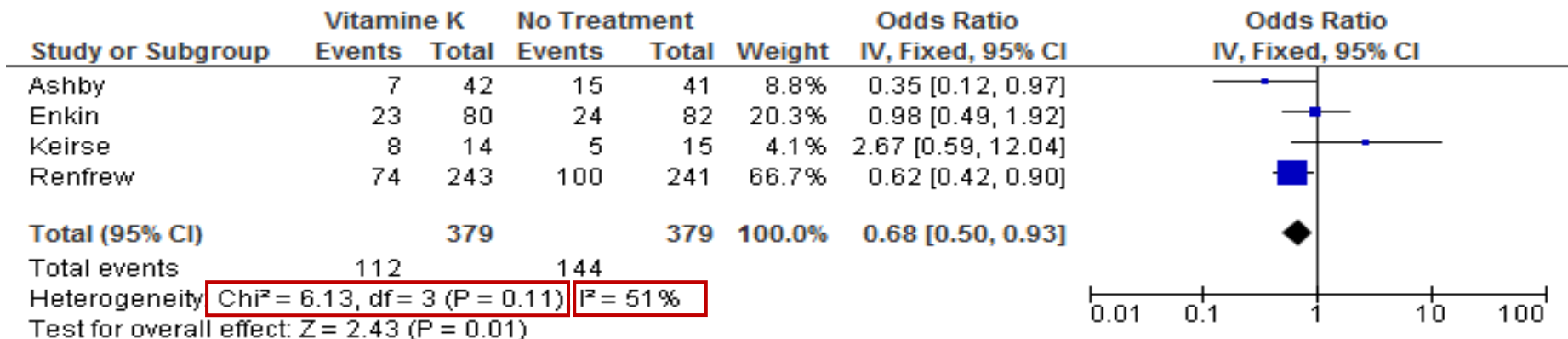
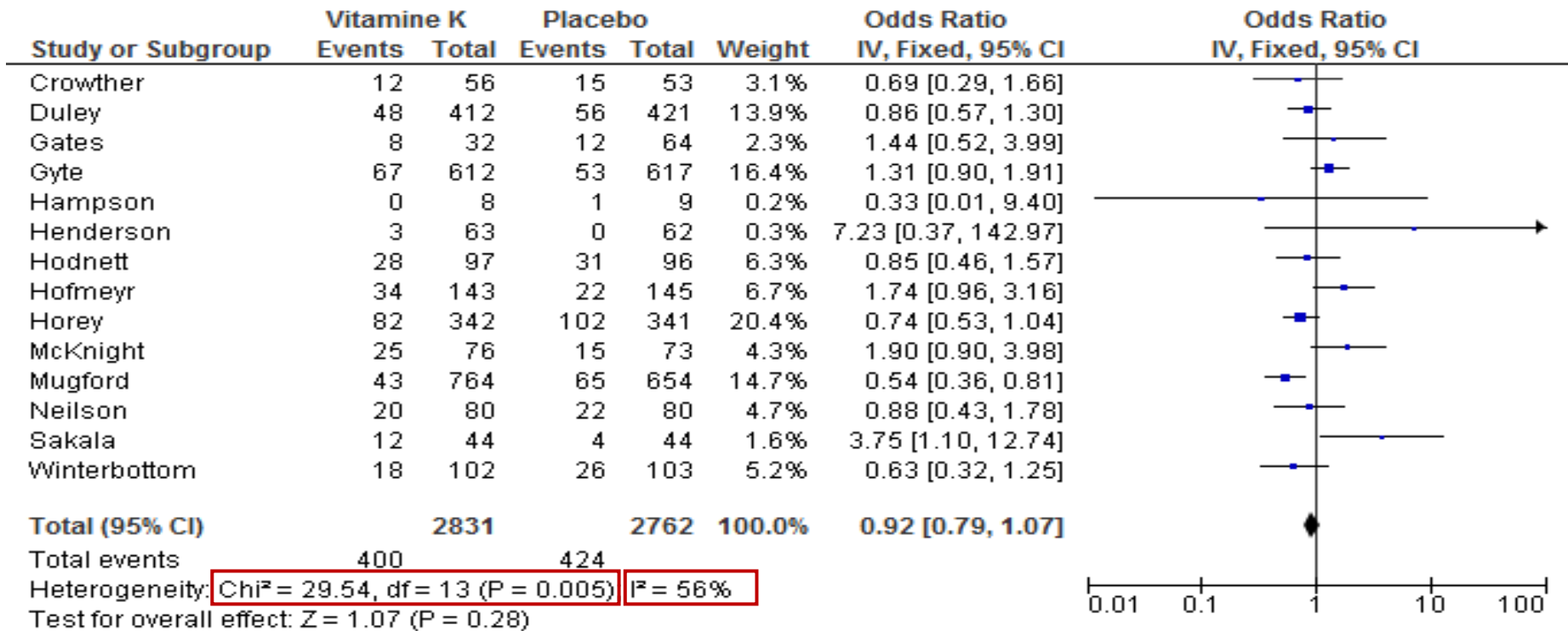
# Estimation of tau-square

- Estimate the heterogeneity variance  $\tau^2$  from the Q-test (method of moments/DL estimator) :

$$\tau^2 = \frac{Q - (k - 1)}{\sum w_i - \frac{\sum w_i^2}{\sum w_i}}$$

- We set  $\tau^2 = 0$  if  $Q < (k - 1)$
- Many other ways to estimate the heterogeneity variance exist (e.g. restricted maximum likelihood)
  - under certain conditions perform better than the DL estimator

# Example: Bleeding





# What can we do with heterogeneity?



- Check the data

- Incorrect data extraction; unit of analysis errors (e.g. with crossover trials, cluster randomized trials, counts)



- Try to bypass it

- Change effect measure

- Encompass it

- Random effects meta-analysis

- Explore it

- Subgroup analysis  
Meta-regression

- Resign to it

- Do no meta-analysis

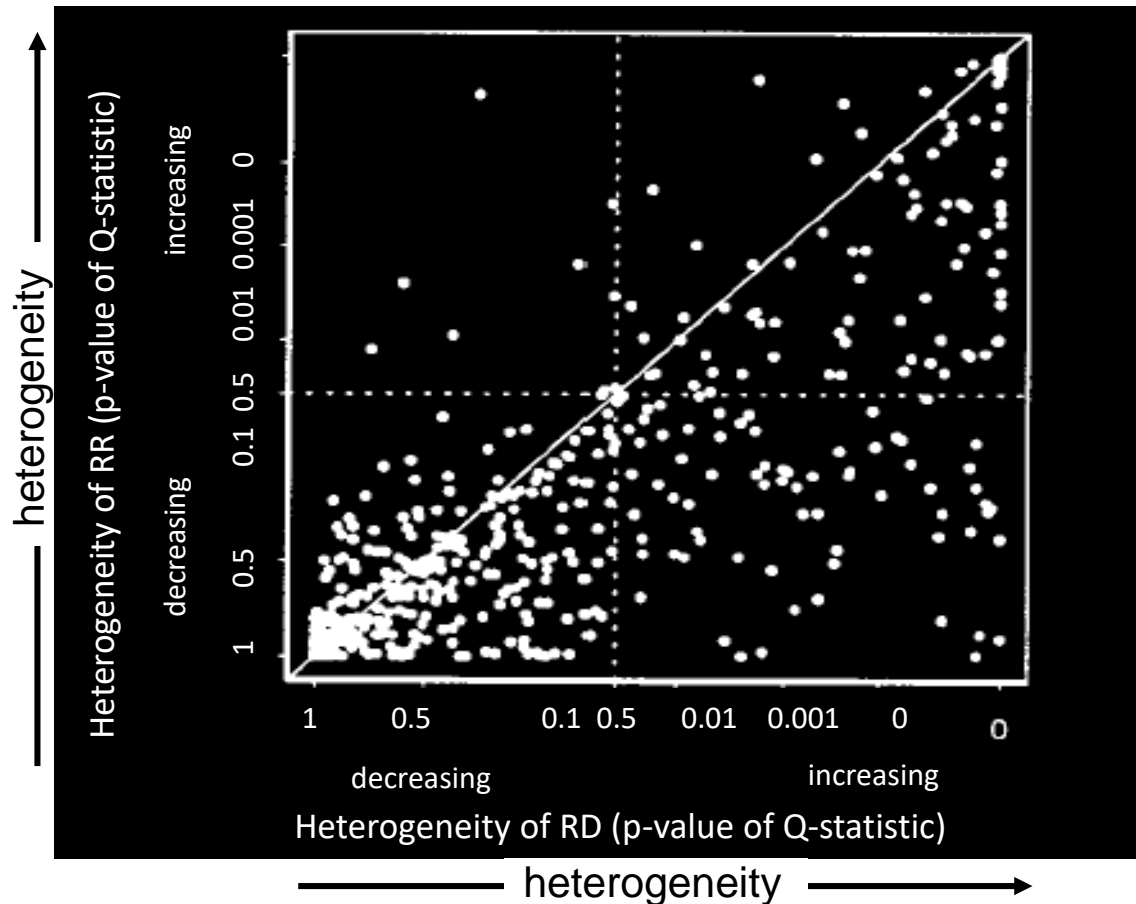
- Ignore it

- Don't do that!

# Heterogeneity of effect measures

## Empirical evidence

- Ratio measures (RR and OR) considerably less heterogeneous than difference measures (RD)



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# What not to do!

- Fixed or random effects meta-analysis should be specified *a priori* if possible and not on the basis of the Q test

## What to do:

Think about the question you asked, the included studies etc: do you expect them to be very diverse?

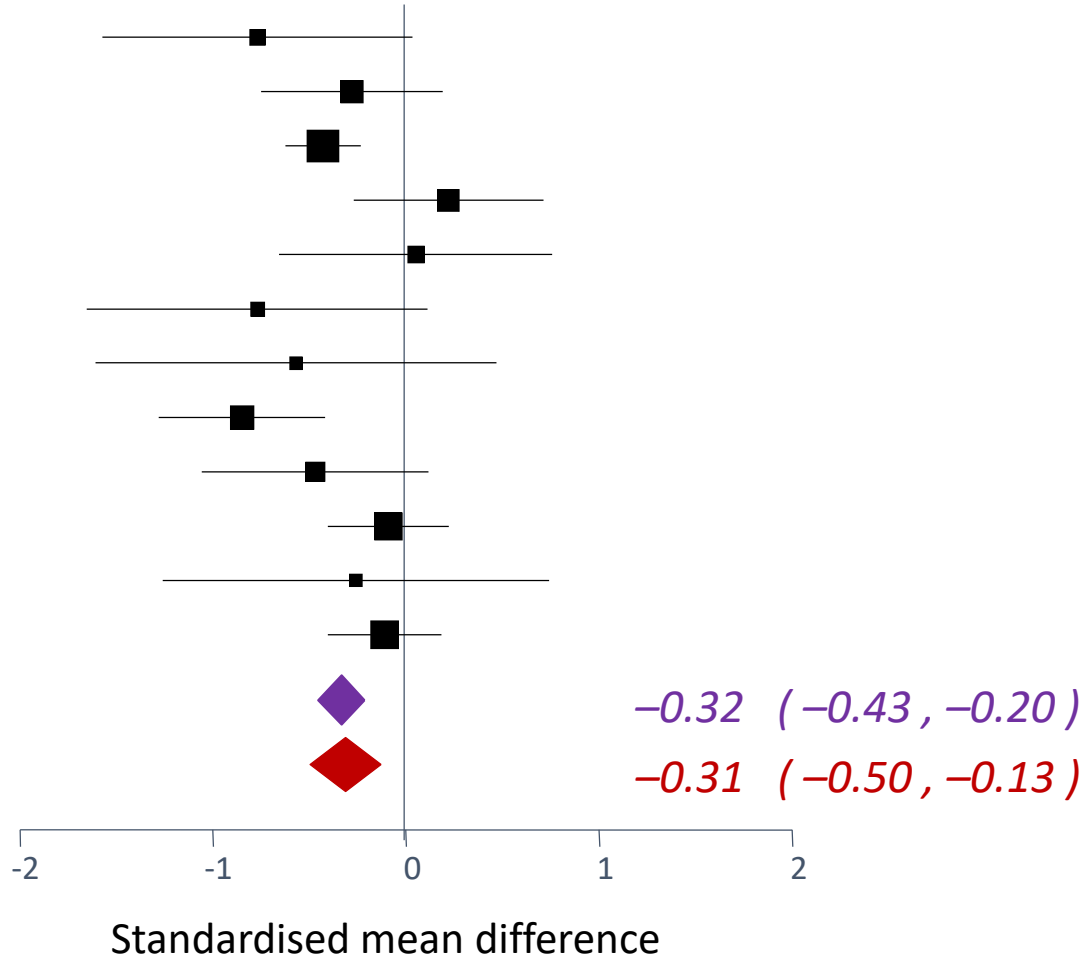
You can apply and present both fixed and random effects

# Fixed vs. random effects

- Fixed effect model is often **unrealistic**
- Random effects model **difficult to interpret**
- Fixed and random effects inverse-variance meta-analyses may
  - be **identical** (when  $\tau^2 = 0$ )
  - give **similar** point estimate, **different** confidence intervals  
*(the 95% CI from FE should fall within the 95% CI from the RE)*

# Example: Opioids for breathlessness

Woodcock 1981  
Woodcock 1982  
Johnson  
Eiser (A)  
Eiser (B)  
Bruera  
Light  
Chua  
Poole  
Davis  
Leung  
Noseda



# Fixed vs. random effects

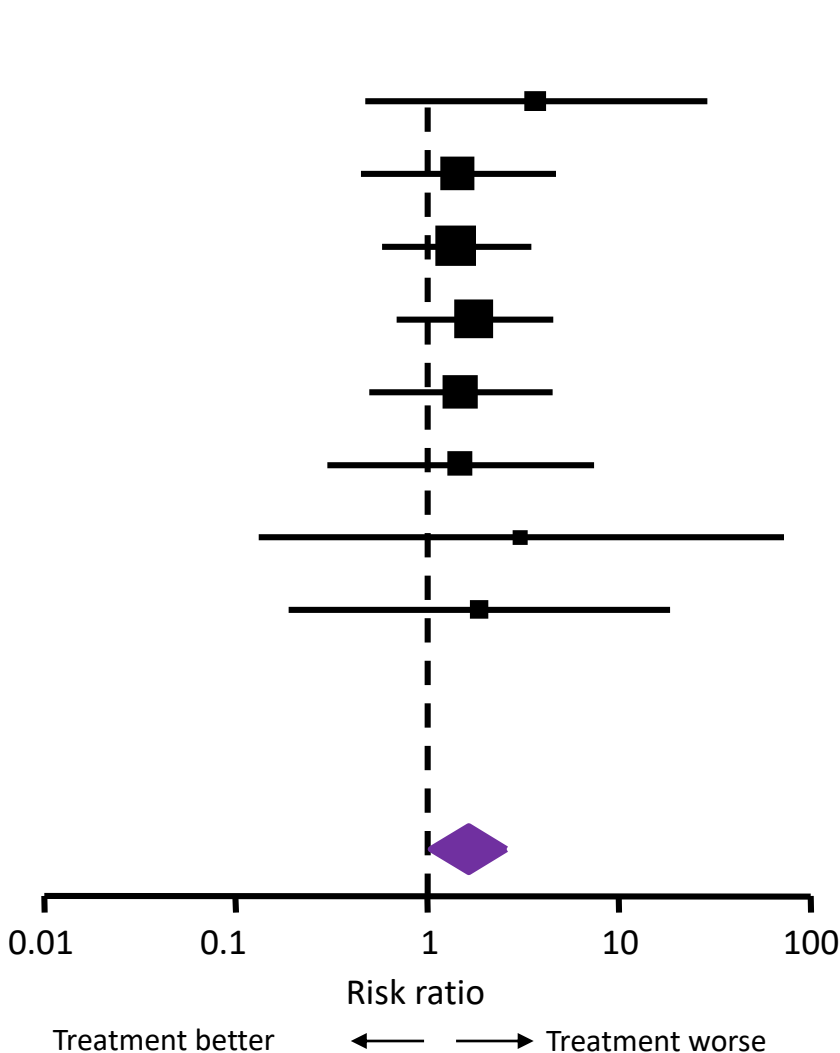
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*(the 95% CI from FE should fall within the 95% CI from the RE)*
- Random effects analysis **may give spurious results** when effect size depends on precision
  - gives relatively more weight to smaller studies
  - important because
    - **smaller studies may be of lower quality (hence biased)**
    - **publication bias may result in missing smaller studies**

# Interpreting random effects meta-analysis

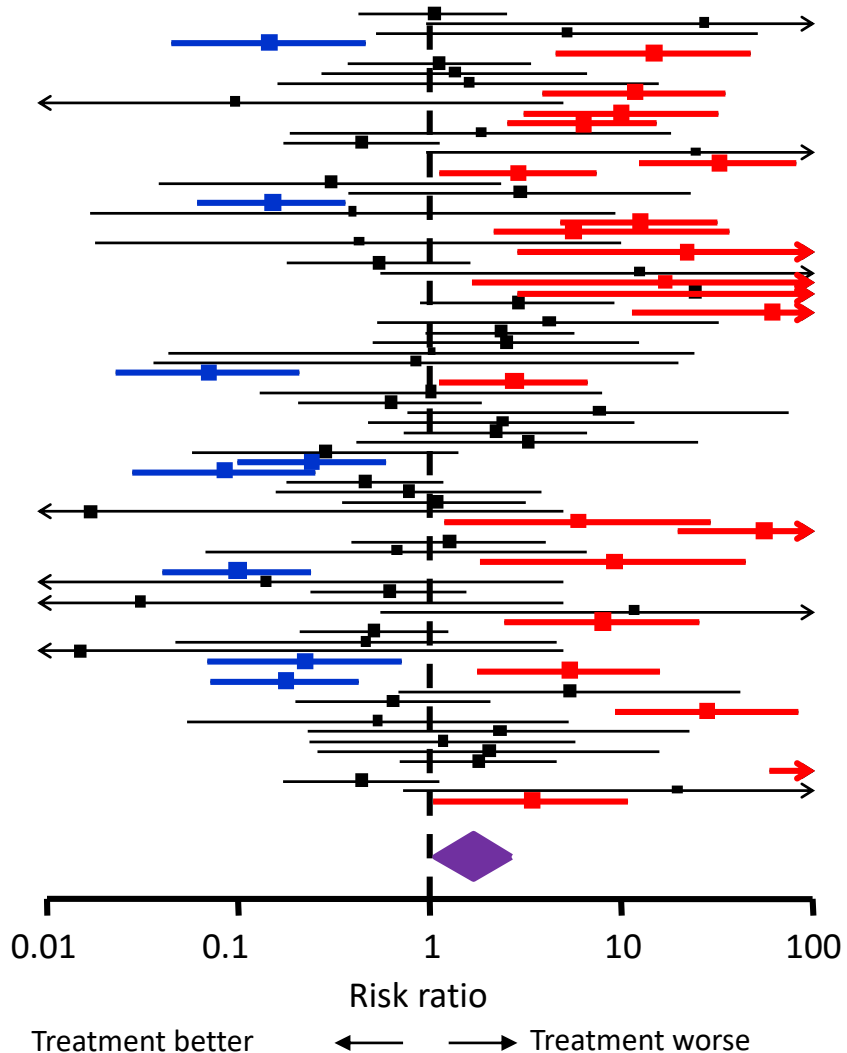
- Random-effects meta-analysis suitable for unexplained heterogeneity
  - Random effects may not explain all the heterogeneity of the data if covariates are responsible
- Conventionally, inference is focused on the mean of the distribution ( $\hat{\theta}$ )
  - i.e. we report mean and 95% from a meta-analysis
  - This may be misleading...



# Example



Fixed effect meta-analysis:  
1.64 ( 1.04 , 2.58 ) P = 0.03



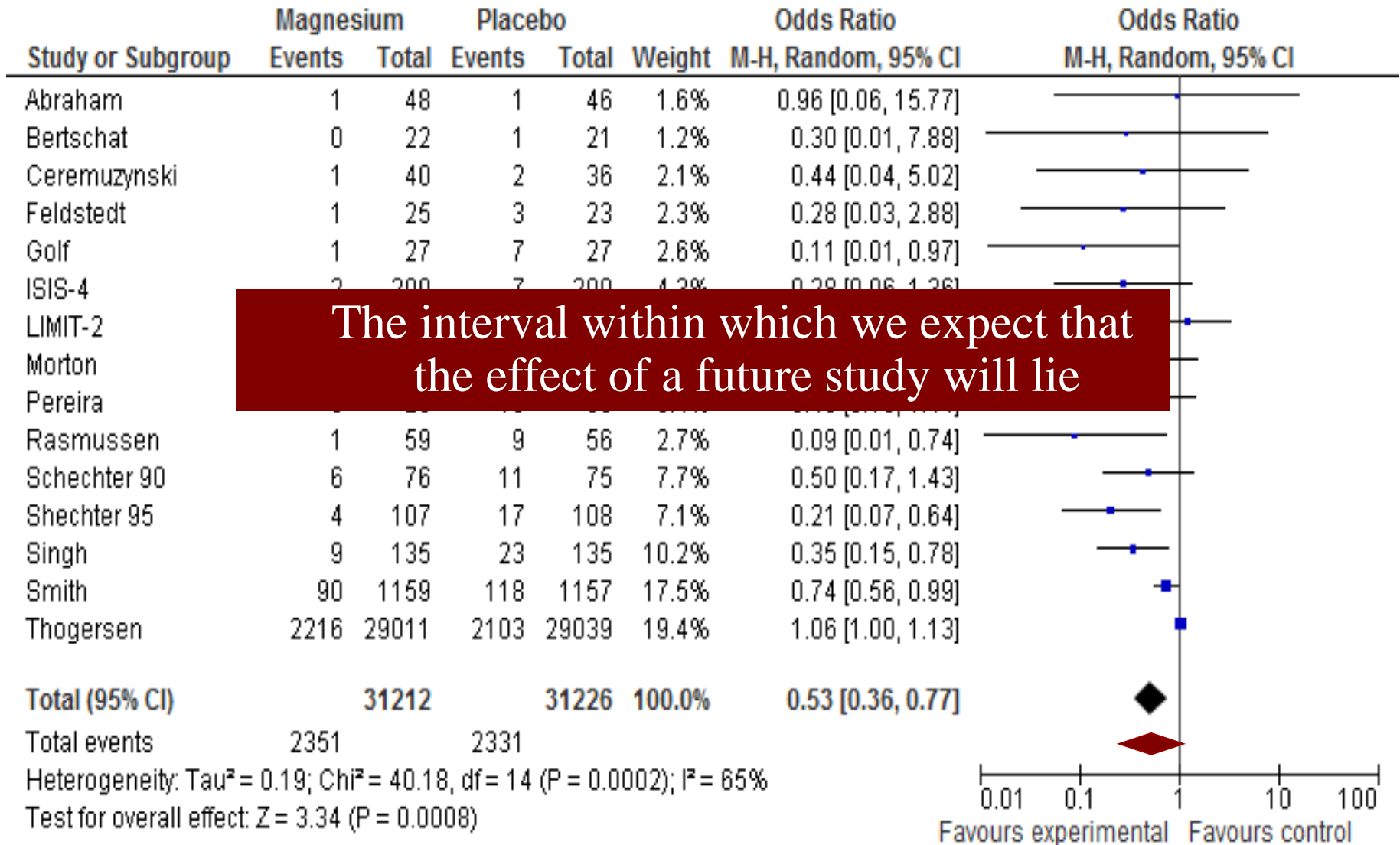
Random effects meta-analysis:  
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# Interpreting random effects meta-analysis

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- Conventionally, inference is focused on the mean of the distribution ( $\hat{\theta}$ )
  - i.e. we report mean and 95% from a meta-analysis
  - This may be misleading...
- Look also at the prediction interval

$$\theta \pm 1.96 \sqrt{se(\theta)^2 + \tau^2}$$
$$\theta \pm t_{0.025, n-1} \sqrt{se(\theta)^2 + \tau^2}$$

# Example



# Interpreting the diamond

- Conventional Interpretations
  1. Statistical significance and direction
  2. Magnitude of the pooled estimate
  3. Width of the confidence interval
- Heterogeneity
  - Too much heterogeneity challenges the meaning of the diamond
- Quality of the included studies

# What can we do with heterogeneity?



- Check the data

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- Try to bypass it

- Change effect measure



- Encompass it

- Random effects meta-analysis



- Explore it

- Subgroup analysis  
Meta-regression

- Resign to it

- Do no meta-analysis

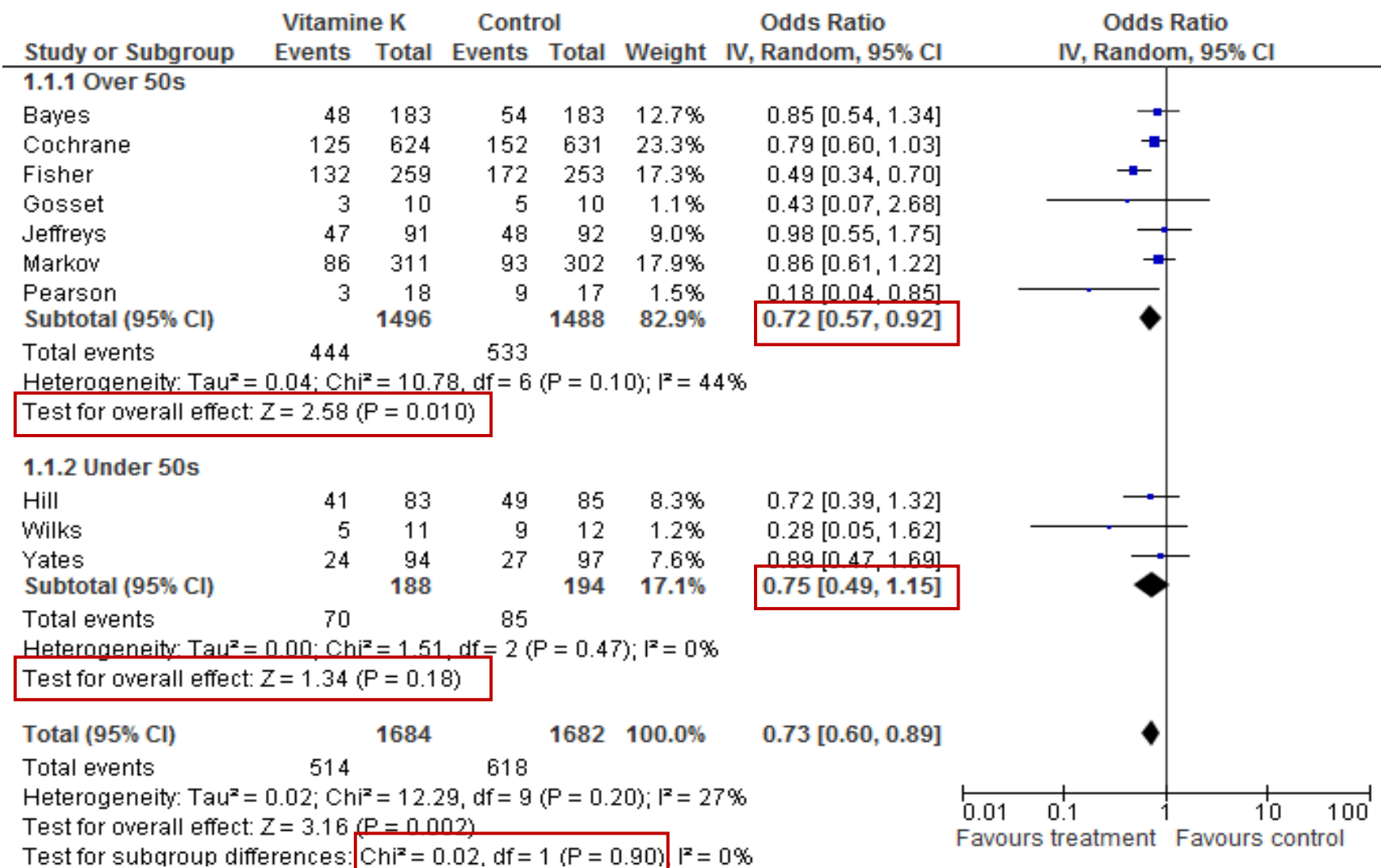
- Ignore it

- Don't do that!

# Exploring heterogeneity

- Characteristics of studies may be associated with the size of treatment effect
- For example,
  - adequacy of allocation concealment
  - average age of patients
  - setting of study
  - dose of drug
- For discrete characteristics, we can use subgroup analyses
- For discrete or continuous characteristics, we can use meta-regression

# Subgroup analysis (example: bleeding)



# Test for differences between subgroups

- $H_0$ : No differences across the  $K$  subgroups
- $H_1$ : There are differences across the  $K$  subgroups

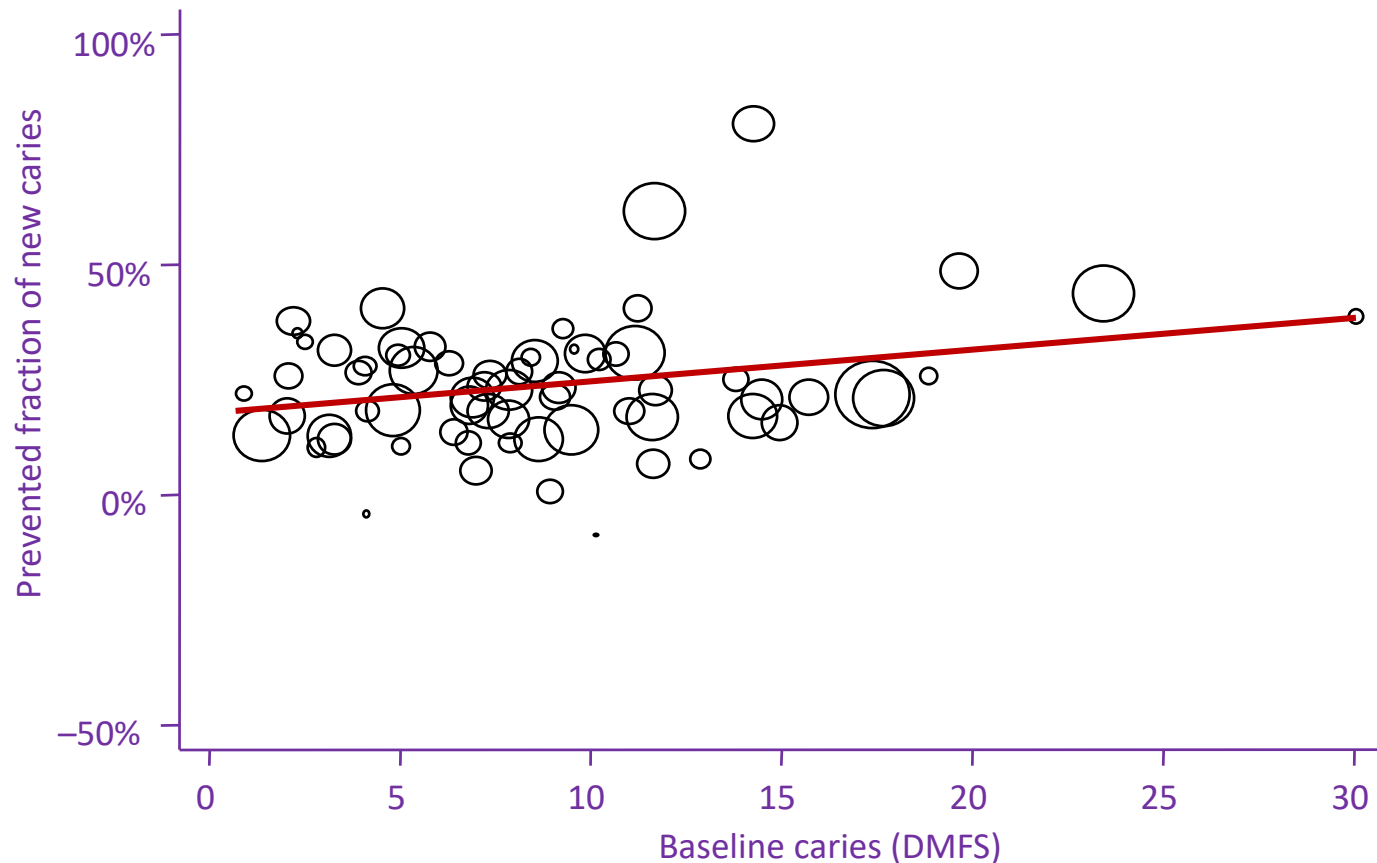
$$Q = Q_{tot} - (Q_1 + Q_2 + \dots + Q_K) \sim \chi_{K-1}^2$$



# Meta-regression

Does effectiveness of toothpaste depend on baseline population levels of caries?

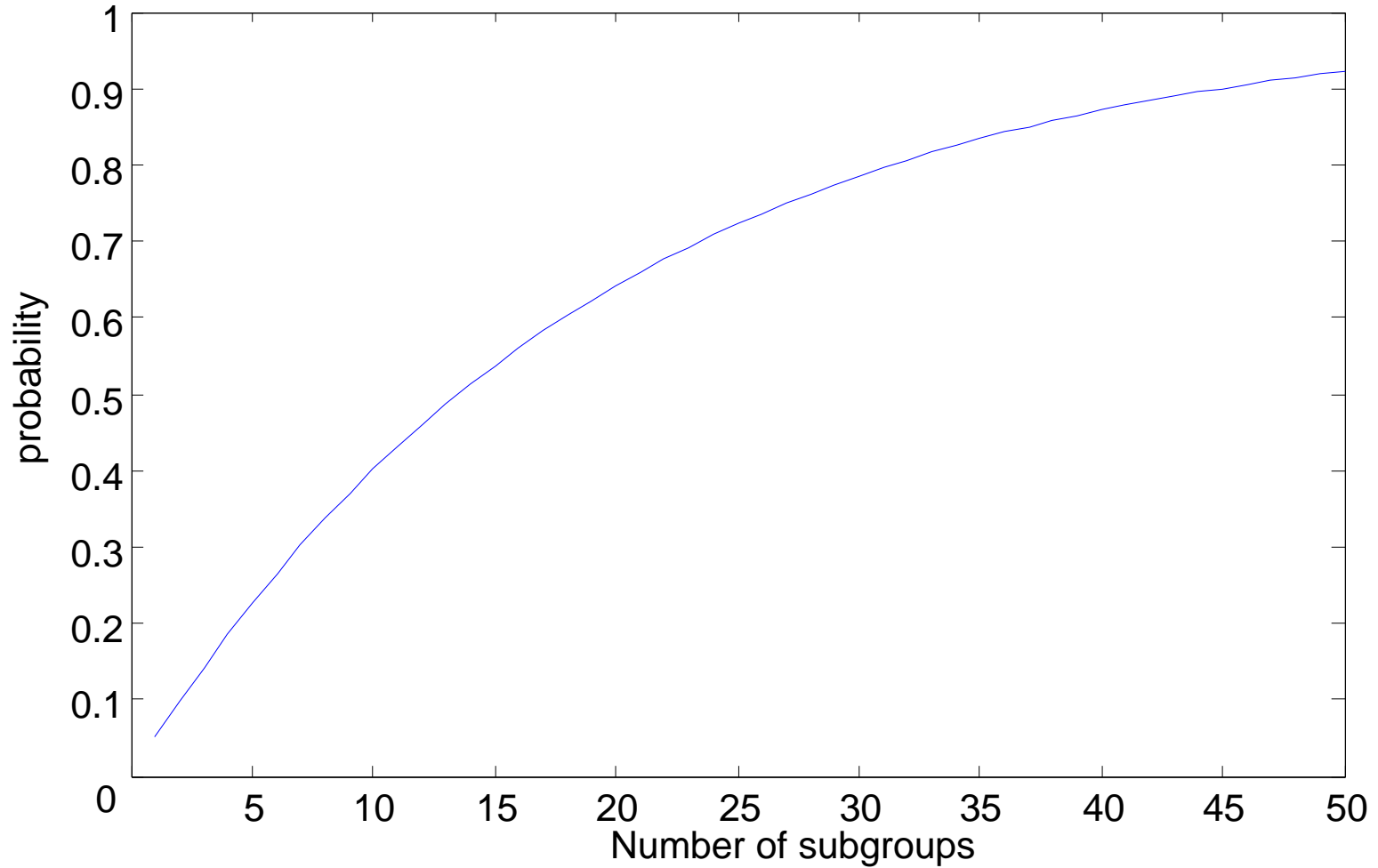
Marinho et al (2003)



# Selecting variables for subgroup analysis and meta-regression

- Specify characteristics in advance
- Select a small number of characteristics
- Ensure there is scientific rationale for investigating the characteristics
  - beware ‘prognostic factors’
- Make sure the effect of a characteristic can be identified
  - does it differentiate studies?
- Think about whether the characteristic is closely related to another characteristic

# Probability of false positive findings



# Problems using published results

- Limited to what is reported
  - Subgroups are rarely reported in all trials
- Limited to “trial-level” characteristics
  - Things that vary between studies; constant within studies
    - Drug dose
    - Treatment duration
- Hard to analyze “participant-level” characteristics
  - Varying between patients in a trial
    - Age
    - Disease severity
  - Rarely reported
  - Using averages (average age, proportion of men) is biased

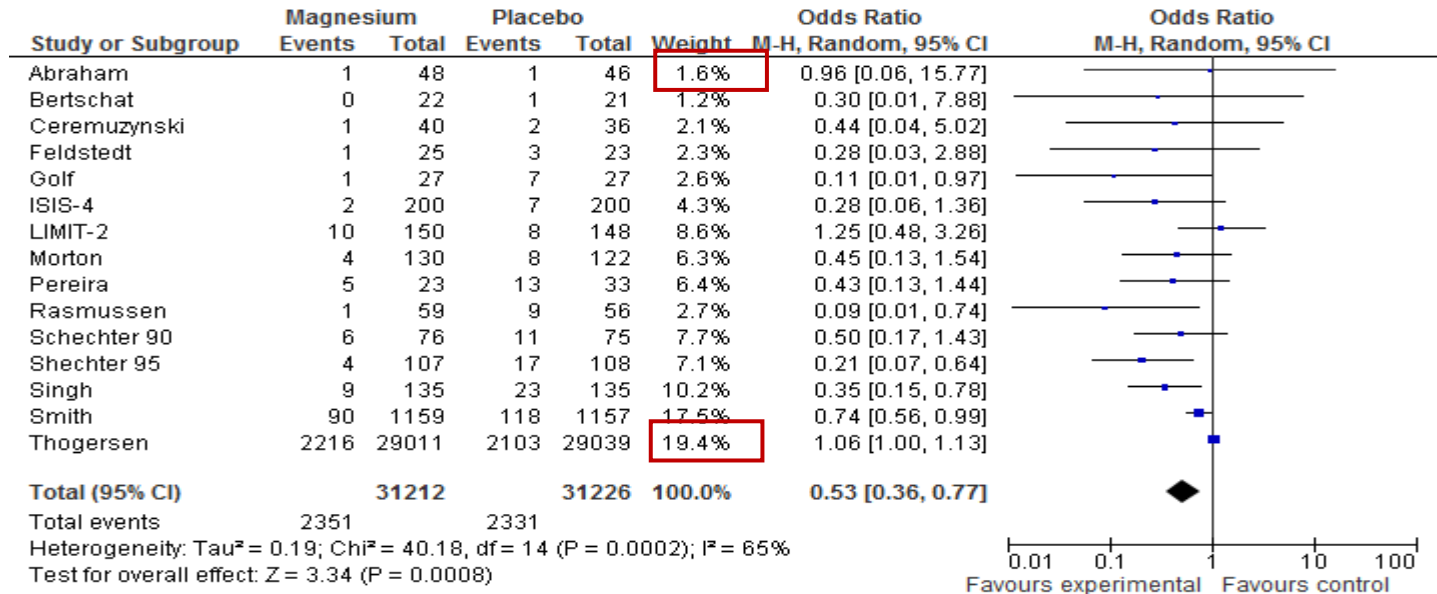
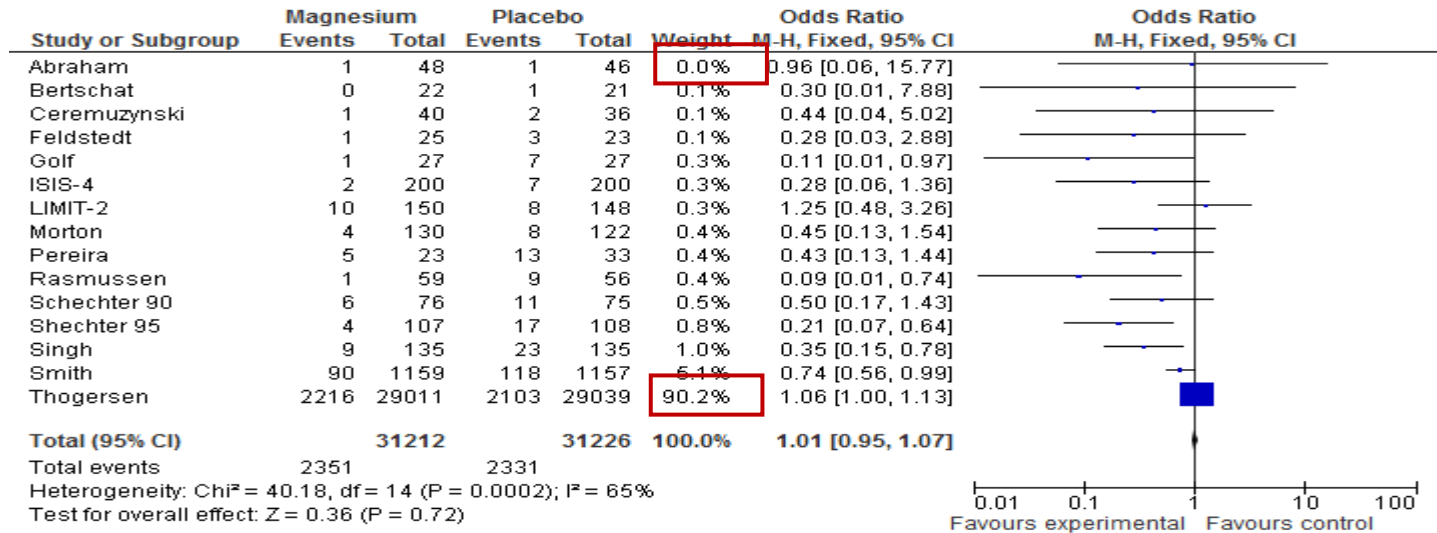
# Using Individual Participant Data

- Obtain all the “raw” data for all participants of all trials
- Gives full data on all characteristics of interest for every participant
  - Age
  - Sex
  - Drug / dose received
  - Exact nature of condition
- Permits analysis of all characteristics of interest
- Usually analyzed using regression modelling
  - Linear / logistic regression
  - NOT subgroup analysis or meta-regression

# Small-study effects and random effects

Magnesium for acute myocardial infraction

Outcome: Mortality



RE gives less 'contrasted' weights between big and small studies

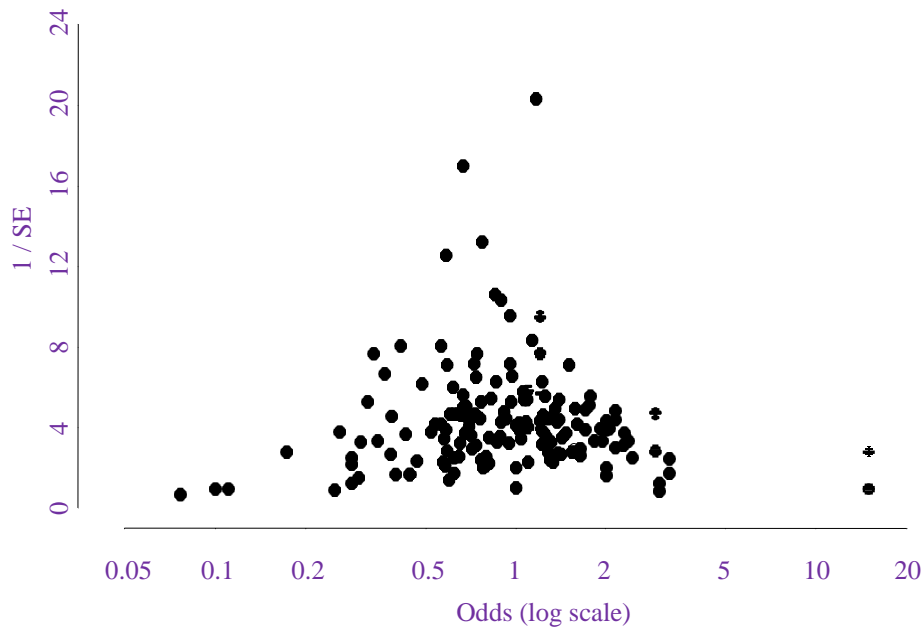
# Small-study effects as a source of heterogeneity

- When the results of your review are related to the size of the study
  - For example smaller studies may give larger treatment effects

# Selective outcome reporting as source of heterogeneity

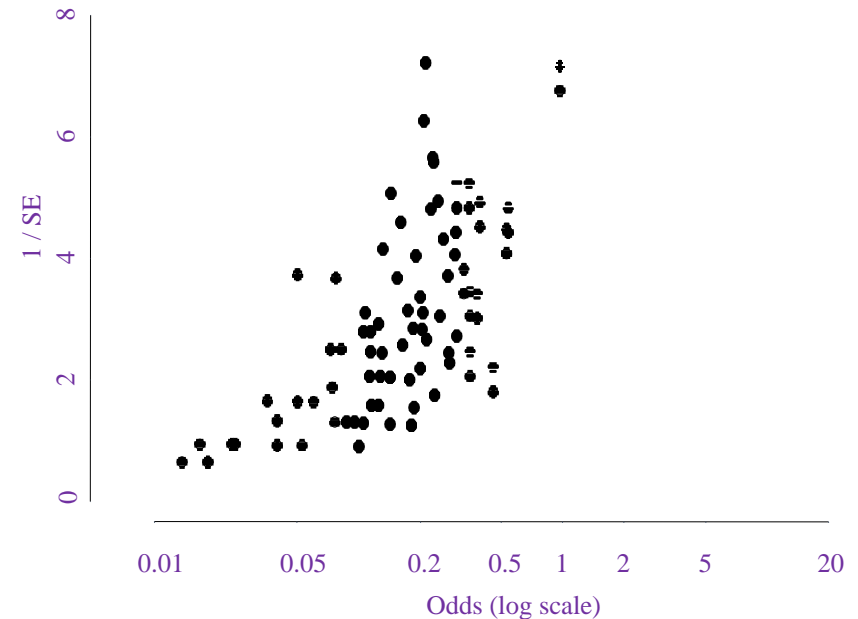
Bown et al. (2002)

Suggests absence of bias



Overall mortality following  
emergency aneurysm repair  
(n=171)

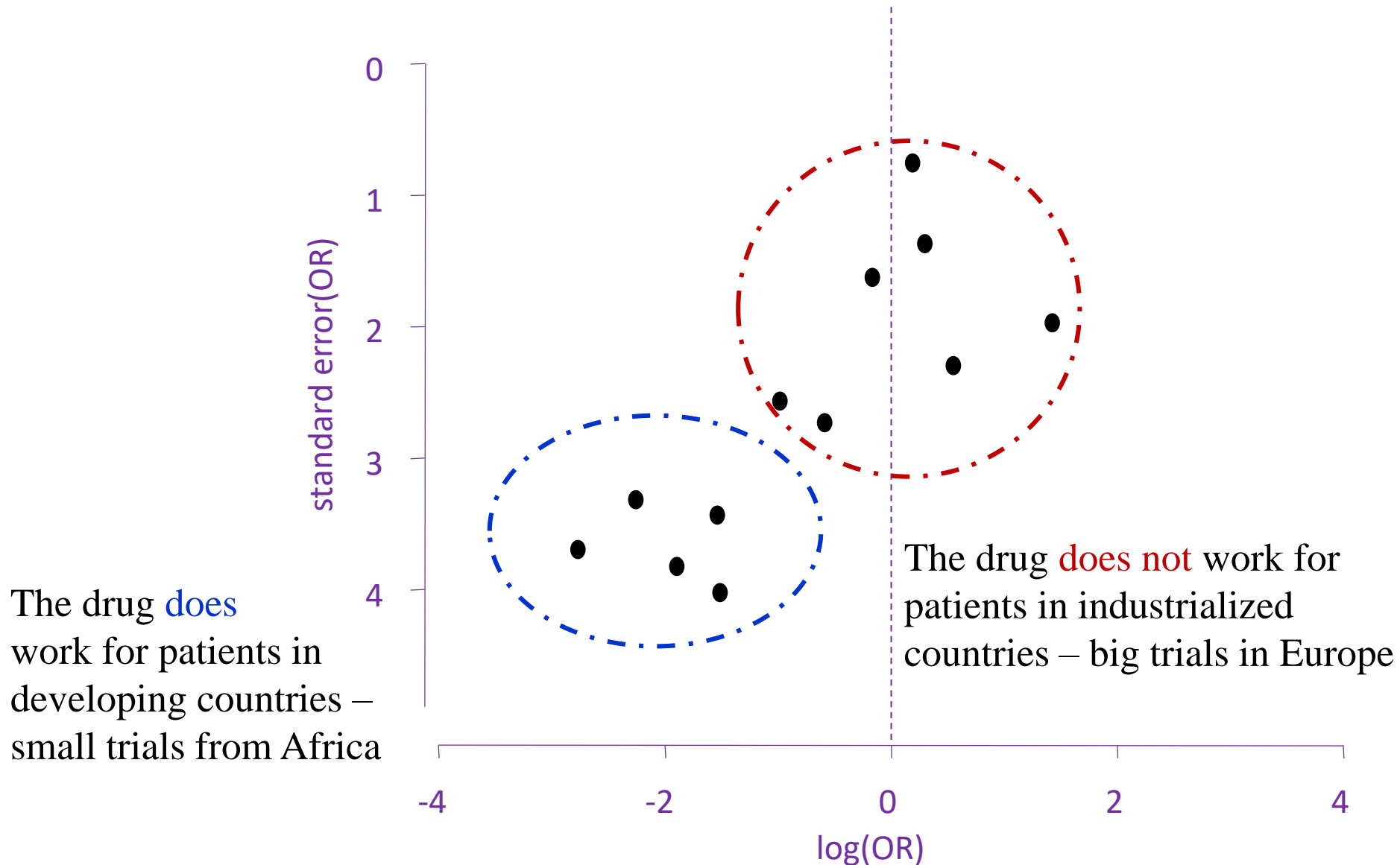
Suggests presence of bias



Mortality in operating theatre  
following emergency  
aneurysm repair (n=77)



# Baseline risk as source of heterogeneity



# What can we do with heterogeneity?

- • Check the data
  - Incorrect data extraction; unit of analysis errors (e.g. with crossover trials, cluster randomized trials, counts)
- • Try to bypass it
  - Change effect measure
- • Encompass it
  - Random effects meta-analysis
- • Explore it
  - Subgroup analysis
  - Meta-regression
- Resign to it
  - Do no meta-analysis
- Ignore it
  - Don't do that!

# Methods available in RevMan

- Estimate of overall effect with CI (fixed effect model)
- Estimate of mean effect with CI (random effects model)
- Test for heterogeneity, with P value
- $I^2$  measure of heterogeneity
- $\tau^2$  heterogeneity variance
- Test for subgroup differences

# Methods not available in RevMan

- Meta-regression
- Random-effects methods that account for the fact that tau-square is estimated
- Prediction intervals
- Individual participant data methods

# References

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