

Explaining heterogeneity

Julian Higgins

School of Social and Community Medicine, University of Bristol, UK

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1. Revision and remarks on fixed-effect and random-effects meta-analysis methods (and interpretation under heterogeneity)

Explaining heterogeneity:

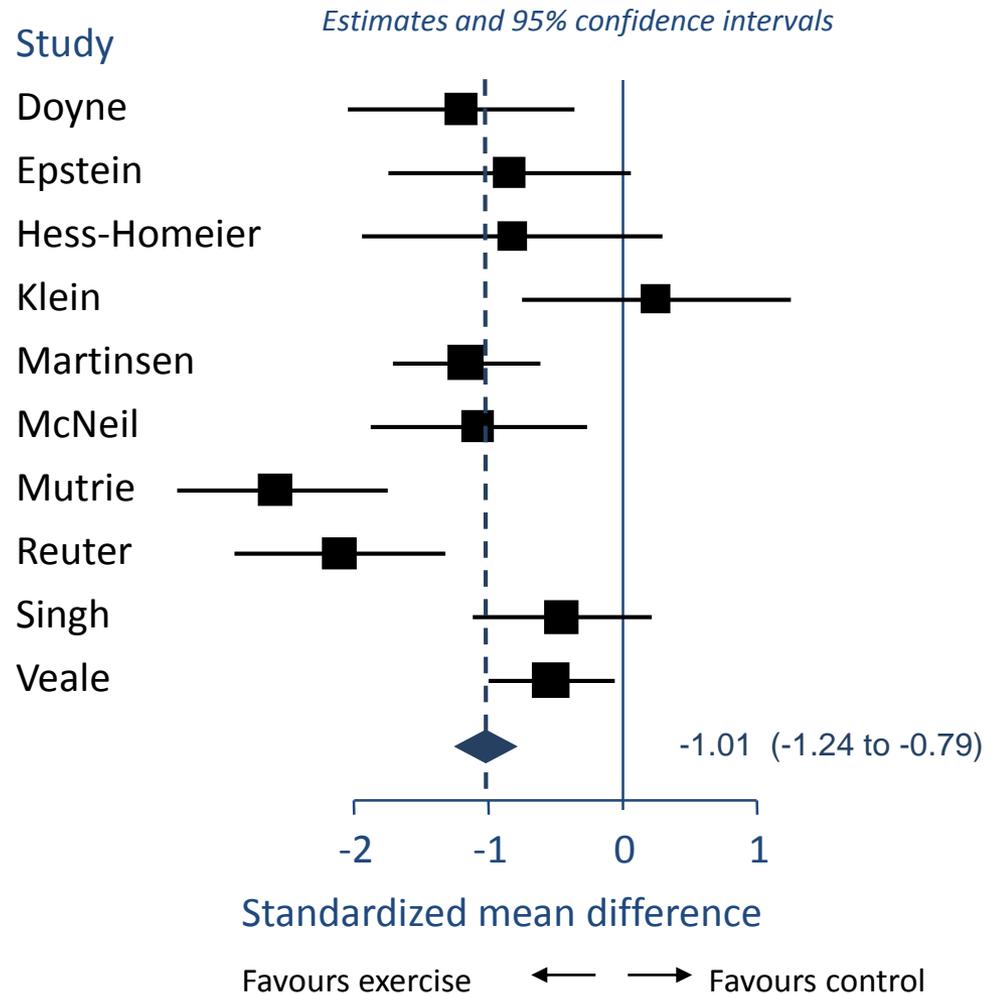
2. Subgroup analysis
 3. Meta-regression
 4. Problems
 5. Closing remarks
-
- Example 1: Trials of exercise for treatment of depression

Part 1: Revision and remarks on fixed-effect and random-effects meta-analysis methods

- Fixed-effect(s) meta-analysis

- weights $W_i = \frac{1}{V_i}$

- (minimise variability of the summary estimate)



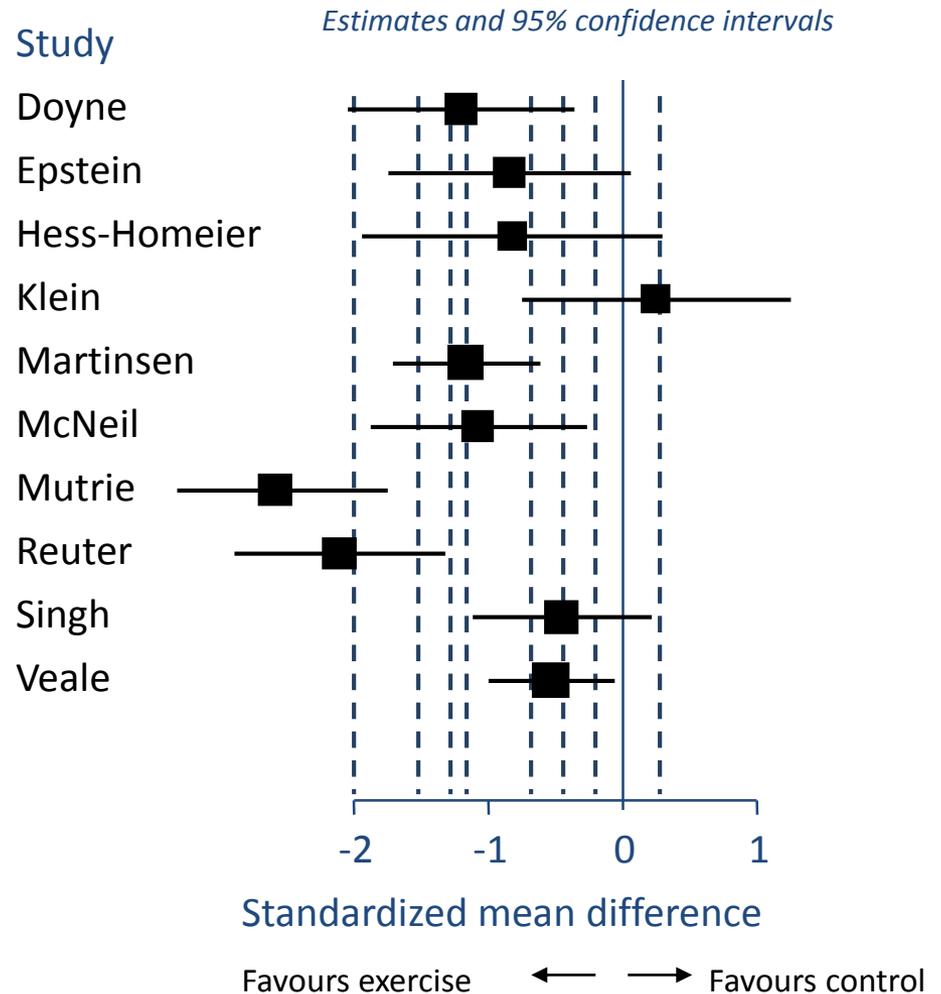
- examine whether effect vary across studies

$$Q = 35.4 \text{ (9 d.f.)}$$

$$(p = 0.00005)$$

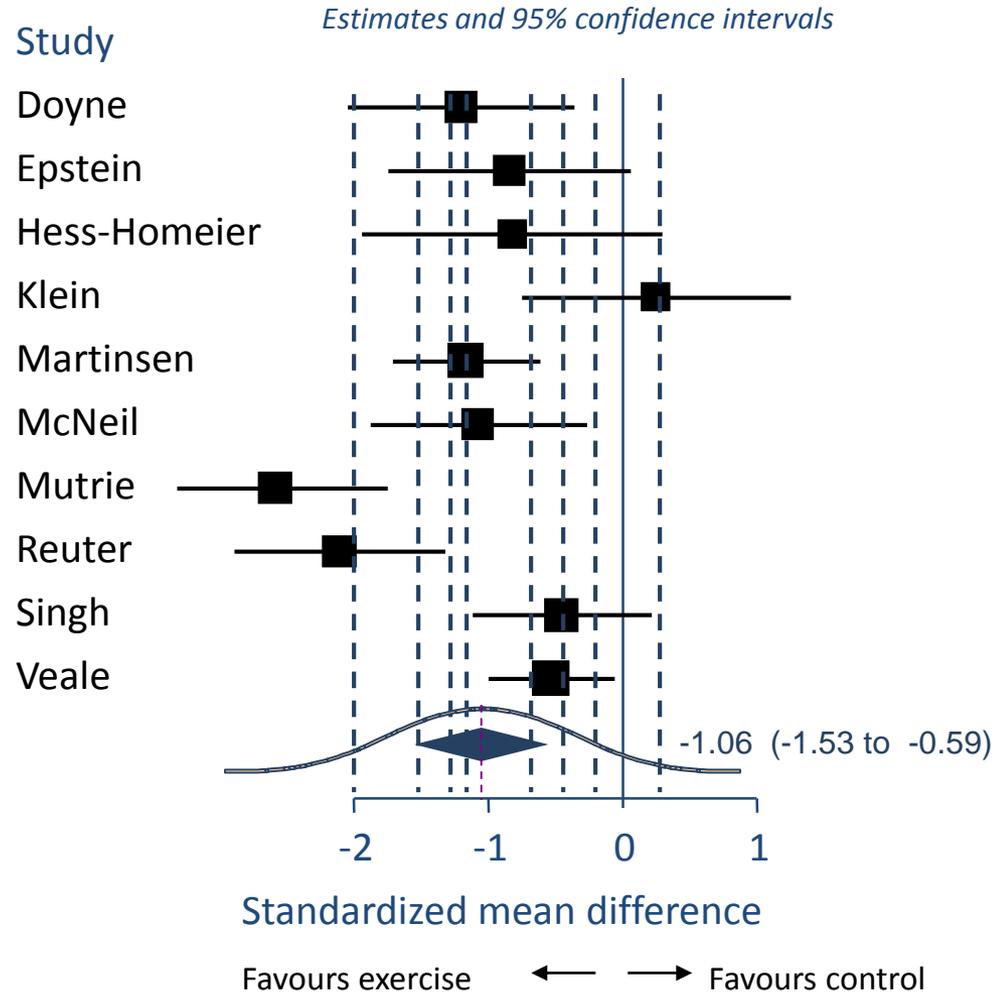
$$\tau_{DL} = 0.64 \text{ (for example)}$$

$$I^2 = 75\%$$

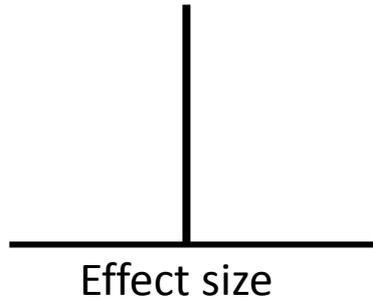


- Meta-analysis under random-effects assumption
 - effect size varies across studies
 - between-study variance = τ^2
 - weights

$$w_i^* = \frac{1}{v_i + \hat{\tau}^2}$$

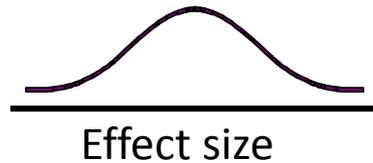


Three simple models for relationships of effect size parameters across studies



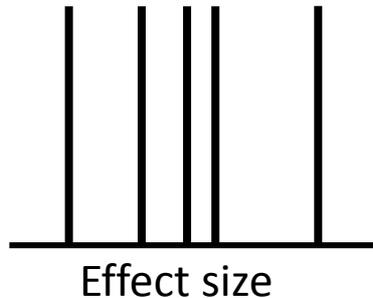
The underlying effect sizes are identical

Common-effect model
(or fixed-effect model)



The underlying effect sizes are **different**
and **related** through some distribution

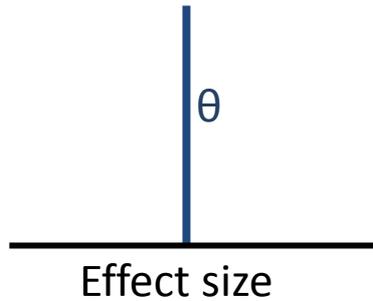
Random-effects model



The underlying effect sizes are **different**
and **unrelated** (independent)

Fixed-effects model

Two simple *methods* for combining effect size *estimates* across studies

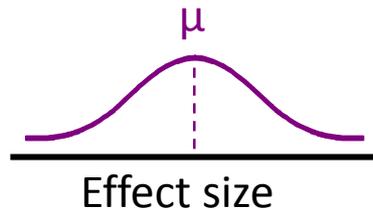


Common-effect model

$$\hat{\theta} = \frac{\sum w_i y_i}{\sum w_i}$$

Estimate of: θ

Variance: $\frac{1}{\sum w_i}$ $w_i = \frac{1}{s_i^2}$

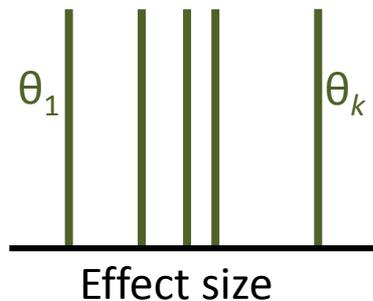


Random-effects model

$$\hat{\mu} = \frac{\sum w_i^* y_i}{\sum w_i^*}$$

Estimate of: μ

Variance: $\frac{1}{\sum w_i^*}$ $w_i^* = \frac{1}{s_i^2 + \hat{\tau}^2}$



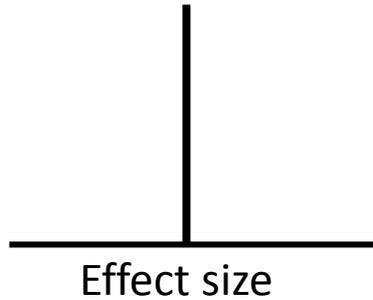
Fixed-effects model

$$\hat{\phi} = \frac{\sum w_i y_i}{\sum w_i}$$

Estimate of: $\frac{\sum w_i \theta_i}{\sum w_i}$

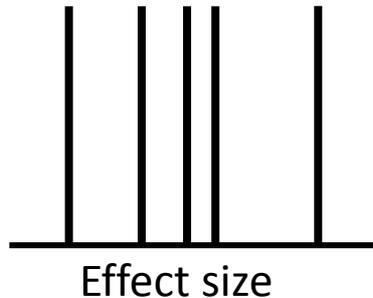
Variance: $\frac{1}{\sum w_i}$

Three simple models for relationships of effect size parameters across studies



Common-effect model
(or **fixed-effect** model)

Complication is that the fixed-effect meta-analysis **method** can be interpreted under either of these **models**



Fixed-effects model

Aside: an interpretation of the weighted average under fixed-effects model

$$\mu = \frac{\sum p_i r_i \theta_i}{\sum p_i r_i}$$

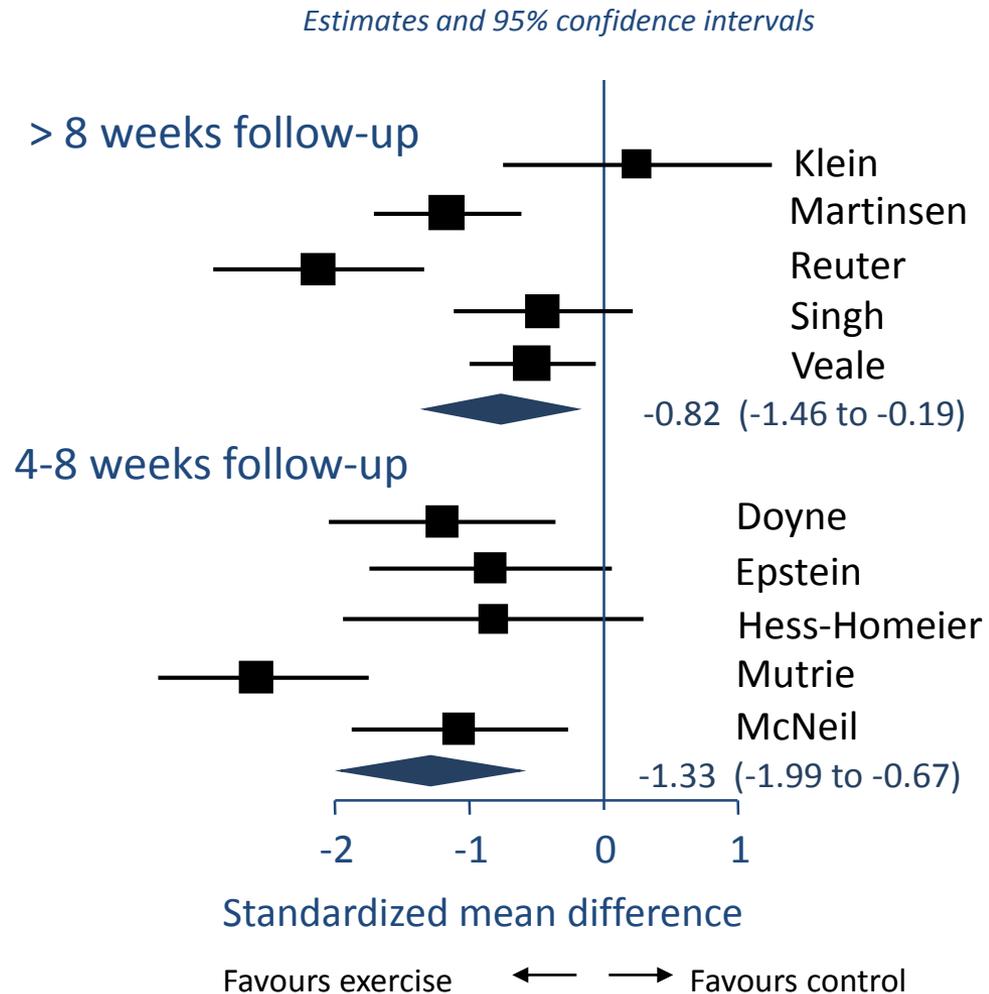
- A weighted average of the study-specific effects seen in the individual study populations, where the weight for study population i is proportional to both the number of participants selected (p_i) and to how much information is contributed per subject (r_i)
- No assumption of homogeneity is required
- But heterogeneity is ignored in the analysis so inference on this is required
 - this inference is statistically independent
- The empirical weights are poor estimates of $p_i r_i$ unless the studies are large

- Variation beyond that expected by chance alone
 - **Observed result = true effect + bias + random error**
- True effects vary due to ‘clinical diversity’
 - **variants in PICO**
- Biases vary due to ‘methodological diversity’
 - **design, conduct, problems**
- In practice, true effects and biases inseparable, so group them together
- Variation in true effect + bias measured by the between-study variance τ^2

In which types of trials does the intervention work best?

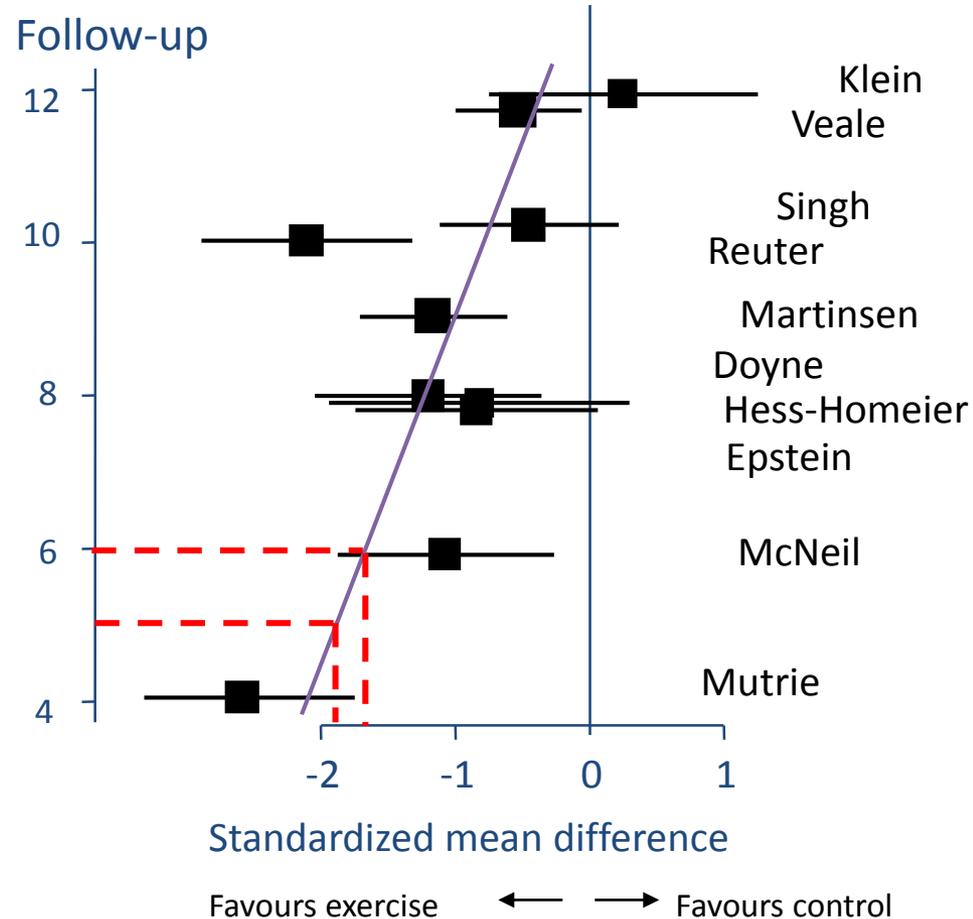
- Characteristics of studies may be associated with the size of treatment effect
 - Does the intervention work better if given for longer?
 - Are smaller odds ratios observed in high-risk populations?
 - Is there a relationship between sample size and effect size (e.g. due to publication bias)?
 - Is inadequate allocation concealment associated with a larger effect estimate?
 - Is A better than B, when they've each only been compared with C? For discrete characteristics, can use subgroup analyses
- We can use subgroup analyses or meta-regression to answer questions like these

- Divide up the studies
 - e.g. by duration of trial
- Test for subgroup differences:
 - can apply Q test to subgroup results
 - here, $P = 0.28$



- Examine heterogeneity
- Predict effect according to length of follow-up
- *Difference in SMDs = 0.18* (95% CI 0.02 to 0.34):
i.e. SMD decreases by 0.2 for each extra week of follow up

Estimates and 95% confidence intervals

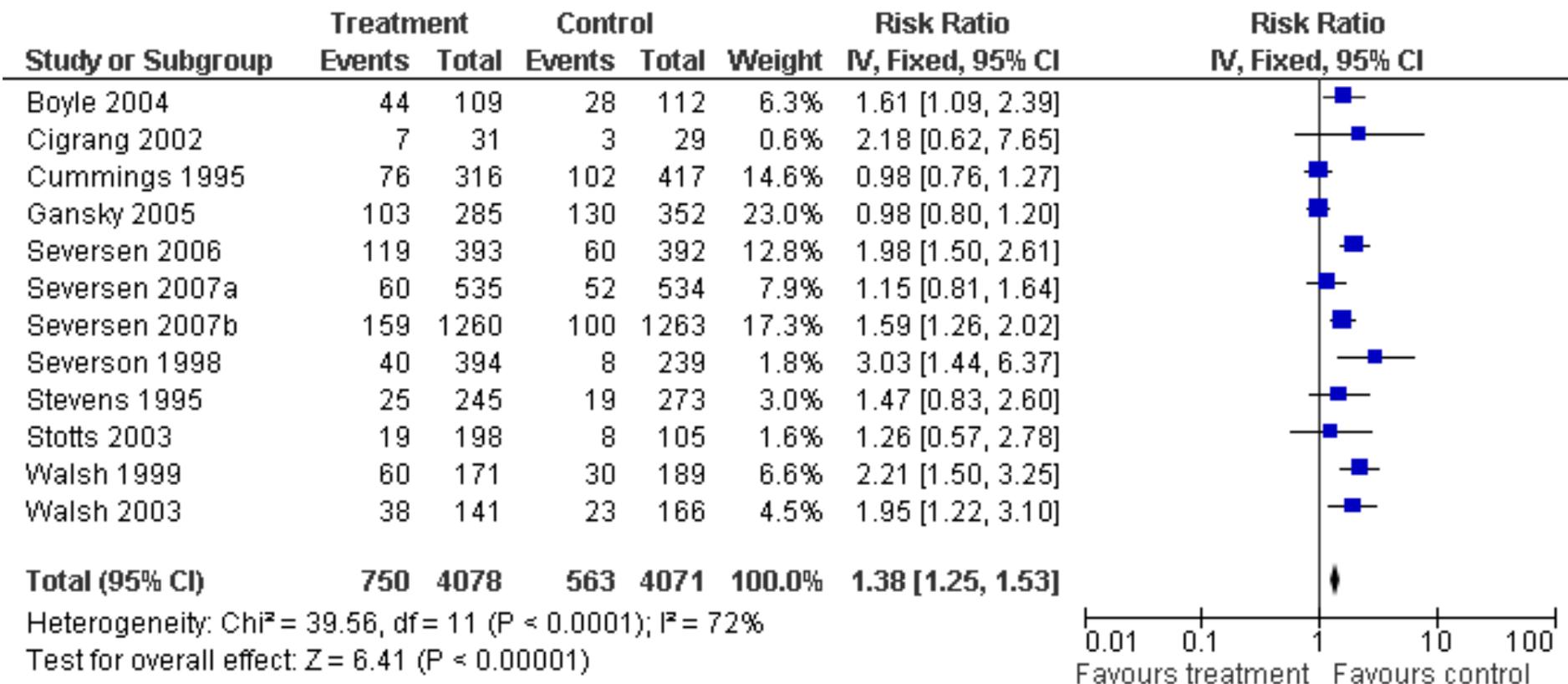


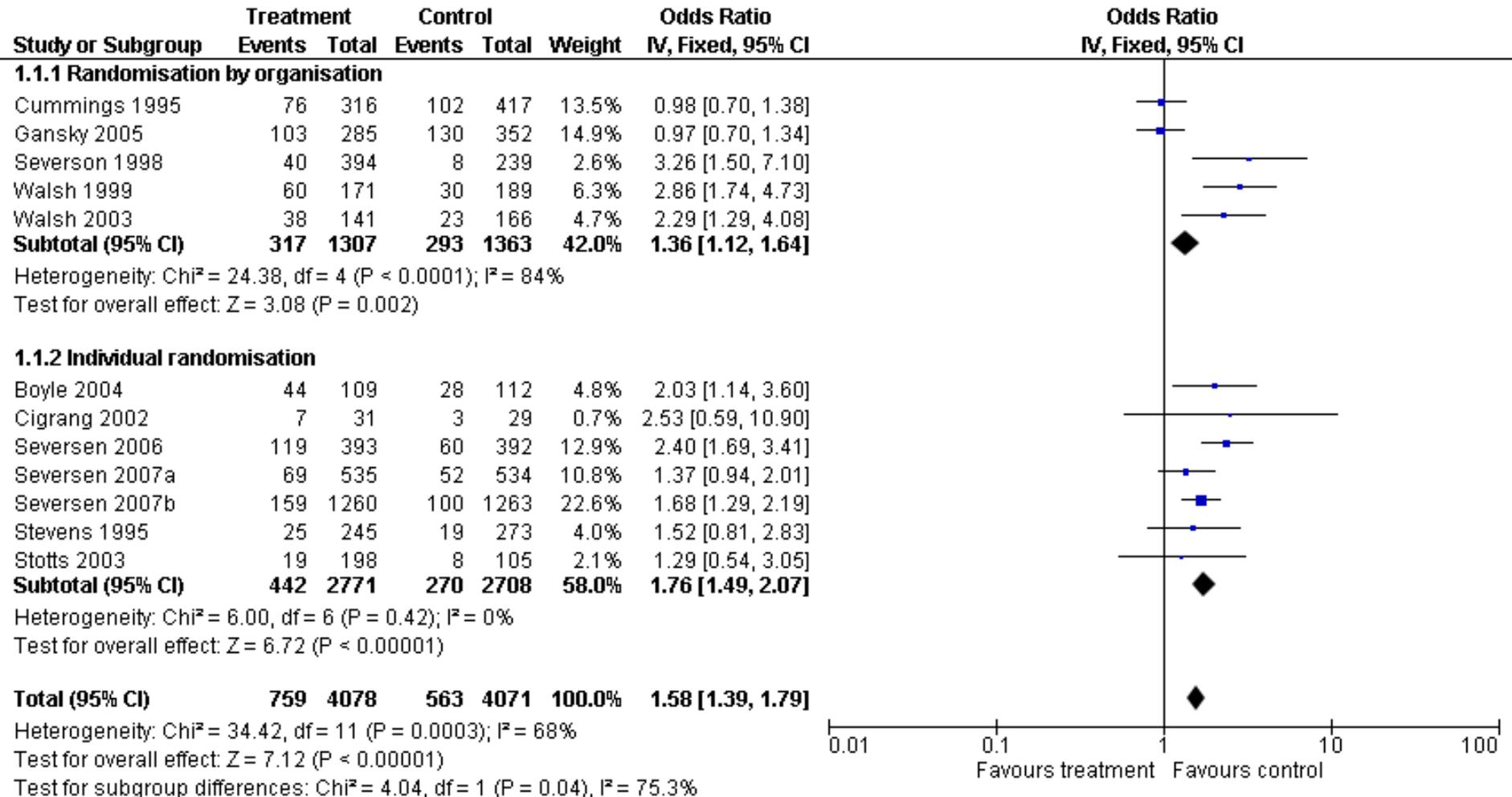
Part 2: Subgroup analysis

- Split data into subgroups:
 - Subsets of patients
 - Subsets of studies
- Do separate meta-analyses for each
- Subgroup analyses done for three reasons
 - To answer multiple questions (one for each subset of studies)
 - To examine whether the effect is different in different subgroups (comparing subgroups)
 - To assess whether restricting analysis to one subgroup changes the conclusion (sensitivity analysis)

Behavioural interventions for smokeless tobacco use cessation

Abstinence from all tobacco use (where reported) at 6 months or more





- Simple statistics: take the difference in point estimates and add their variances
 - all on the log scale for ratio measures

Subgroup	OR	OR_CI_low	OR_CI_upp	lnOR	lnOR_CI_low	lnOR_CI_upp	lnOR_var
Randomisation by organisation	1.36	1.12	1.64	0.307	0.113	0.495	0.009
Individual randomisation	1.76	1.49	2.07	0.565	0.399	0.728	0.007
	RATIO			DIFF			SUM
Comparison of subgroups	0.773	0.601	0.994	-0.258	-0.510	-0.006	0.016

- Simple method

Q_{all} : chi-squared statistic for all the studies

Q_1, \dots, Q_m : chi-squared statistic for m different subgroups

- Heterogeneity explained by differences between subgroups (Q_{bet}):

$$Q_{\text{bet}} = Q_{\text{all}} - (Q_1 + \dots + Q_m)$$

- Has degrees of freedom

$$\text{df} = m - 1$$

Fixed-effects test for subgroup differences

Study or Subgroup	Treatment		Control		Weight	Odds Ratio IV, Fixed, 95% CI
	Events	Total	Events	Total		
1.1.1 Randomisation by organisation						
Cummings 1995	76	316	102	417	13.5%	0.98 [0.70, 1.38]
Gansky 2005	103	285	130	259	14.9%	0.97 [0.70, 1.34]
Severson 1998	40	394	8	363	2.6%	3.26 [1.50, 7.10]
Walsh 1999	60	171	30	161	6.3%	2.86 [1.74, 4.73]
Walsh 2003	38	141	23	134	4.7%	2.29 [1.29, 4.08]
Subtotal (95% CI)	317	1307	293	1363	42.0%	1.36 [1.12, 1.64]

Heterogeneity: $\text{Chi}^2 = 24.38$, $\text{df} = 4$ ($P < 0.0001$); $I^2 = 84\%$

Test for overall effect: $Z = 3.08$ ($P = 0.002$)

1.1.2 Individual randomisation

Boyle 2004	44	109	28	112	4.8%	2.03 [1.14, 3.60]
Cigrang 2002	7	31	3	29	0.7%	2.53 [0.59, 10.90]
Severson 2006	119	393	60	392	12.9%	2.40 [1.69, 3.41]
Severson 2007a	69	535	5	520	10.8%	1.37 [0.94, 2.01]
Severson 2007b	159	1260	10	1250	22.6%	1.68 [1.29, 2.19]
Stevens 1995	25	245	1	234	4.0%	1.52 [0.81, 2.83]
Stotts 2003	19	198	8	195	2.1%	1.29 [0.54, 3.05]
Subtotal (95% CI)	442	2771	270	2708	58.0%	1.76 [1.49, 2.07]

Heterogeneity: $\text{Chi}^2 = 6.00$, $\text{df} = 6$ ($P = 0.42$); $I^2 = 0\%$

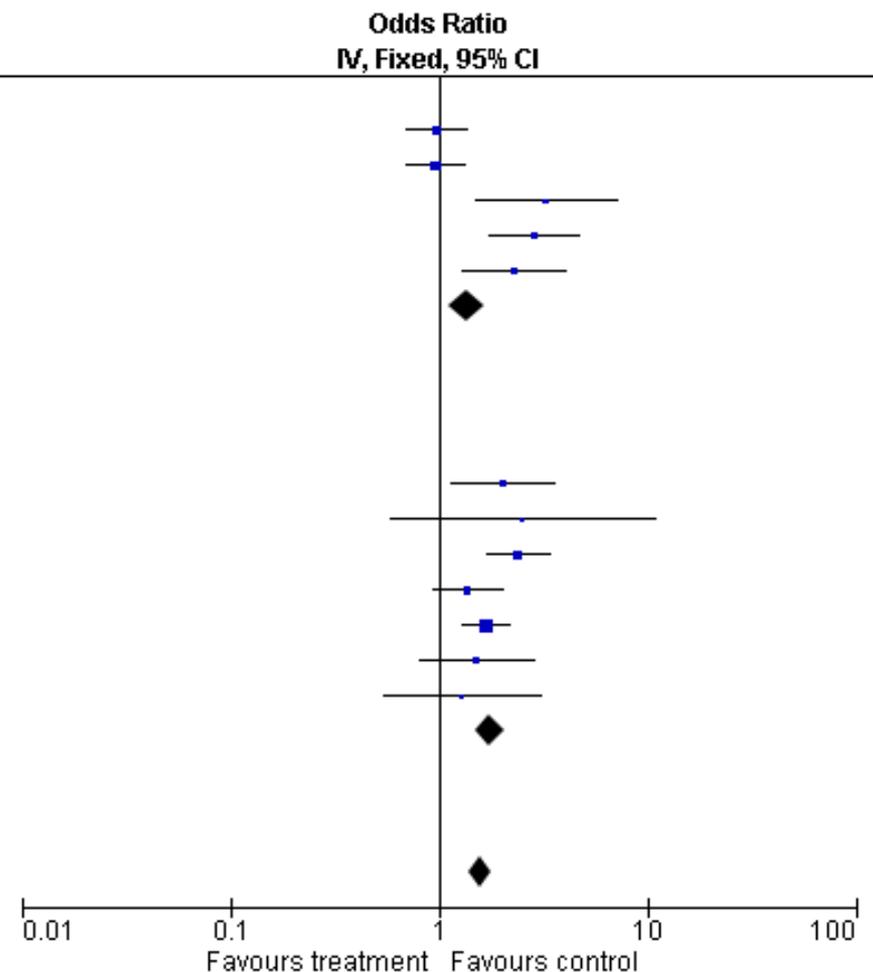
Test for overall effect: $Z = 6.72$ ($P < 0.00001$)

Total (95% CI) **759** **4078** **563** **4071** **100.0%** **1.58 [1.39, 1.79]**

Heterogeneity: $\text{Chi}^2 = 34.42$, $\text{df} = 11$ ($P = 0.0003$); $I^2 = 68\%$

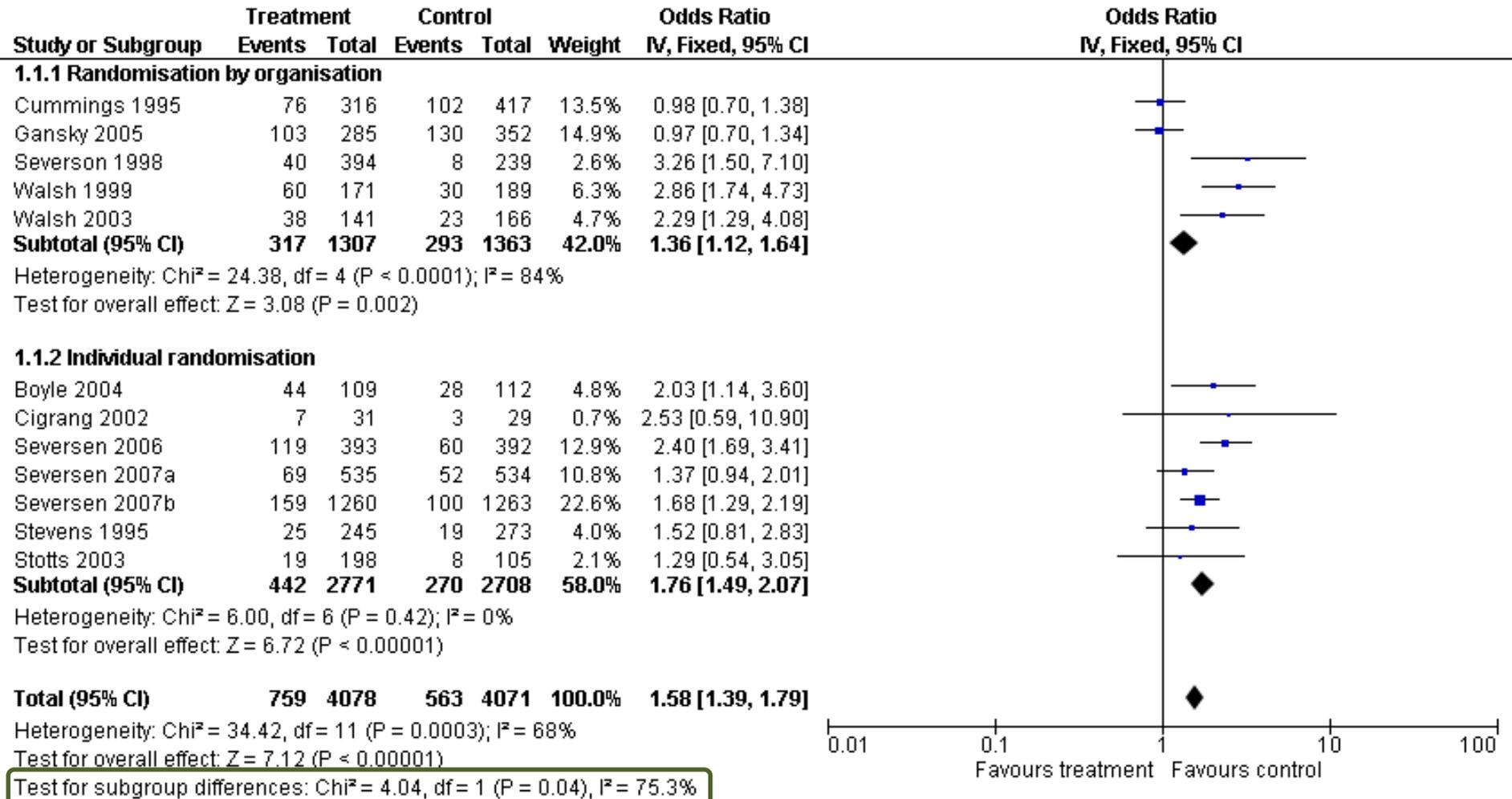
Test for overall effect: $Z = 7.12$ ($P < 0.00001$)

Test for subgroup differences: $\text{Chi}^2 = 4.04$, $\text{df} = 1$ ($P = 0.04$), $I^2 = 75.3\%$

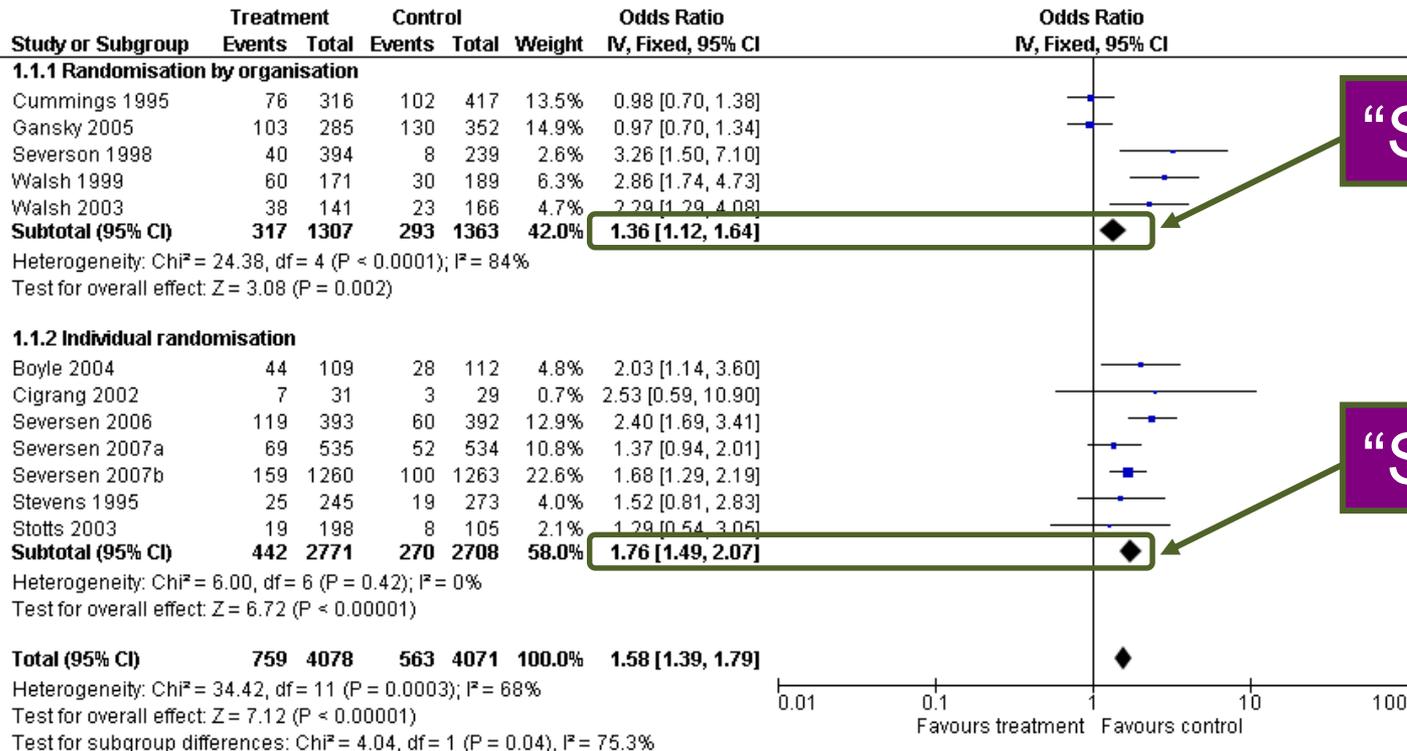


$$\begin{aligned} Q_{\text{bet}} &= 34.42 - (24.38 + 6.00) \\ &= 4.04 \end{aligned}$$

- Two subgroups, so $2 - 1 = 1$ degree of freedom
- Chi-squared value of 4.04 with 1 degree of freedom:
 $P = 0.044$



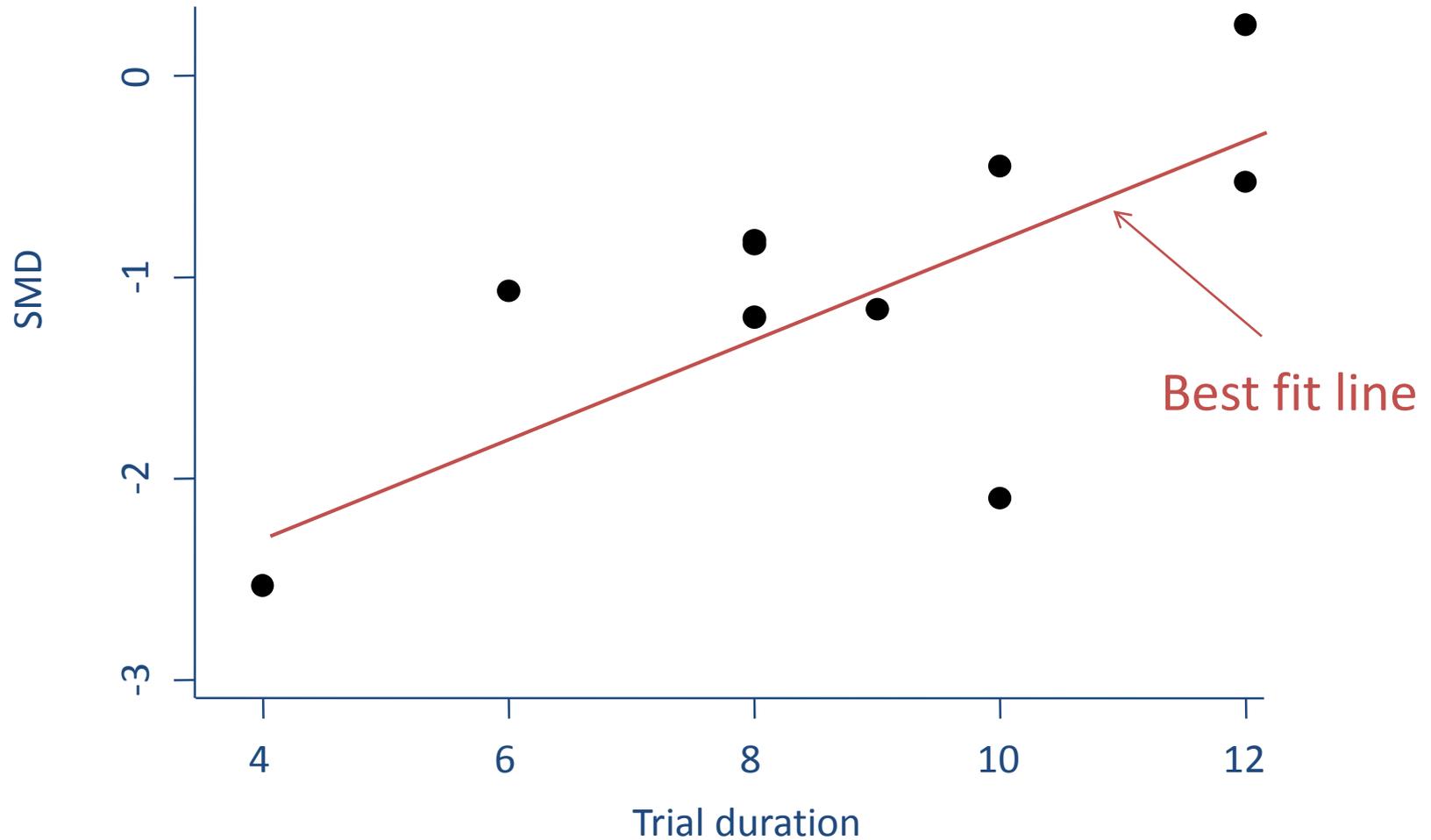
- Just do a test for heterogeneity across the results of all the subgroups
- Fine for any number of subgroups, and any type of analysis within the subgroups



Part 3: Meta-regression

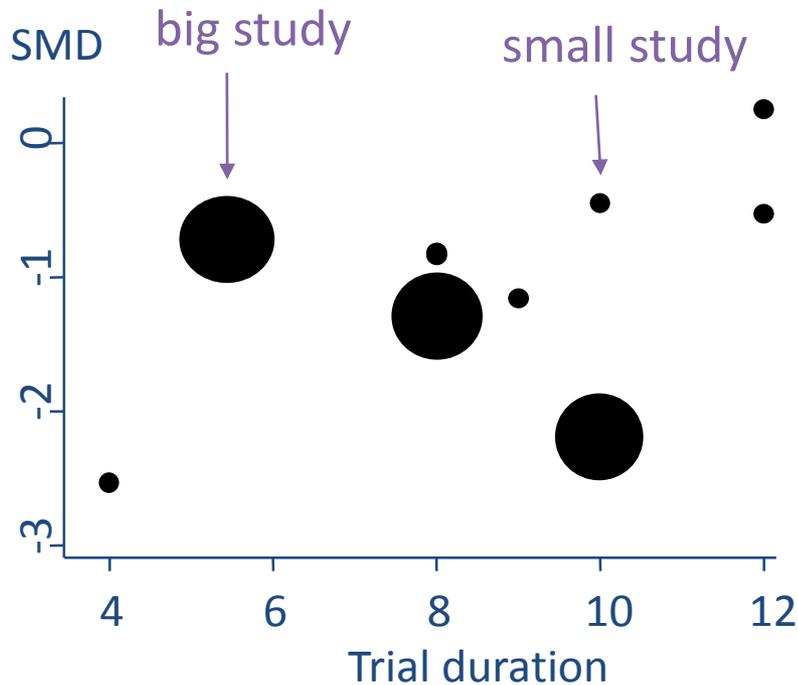
- A general framework for looking at possible explanations for differences in effect sizes across studies
- A test for subgroup differences can be done using meta-regression
- But meta-regression also good for continuous characteristics
- Linear regression
 - outcome variable = effect size (e.g. SMD or logRR)
 - explanatory variable = summary description of study (e.g. duration or location)

Simple linear regression

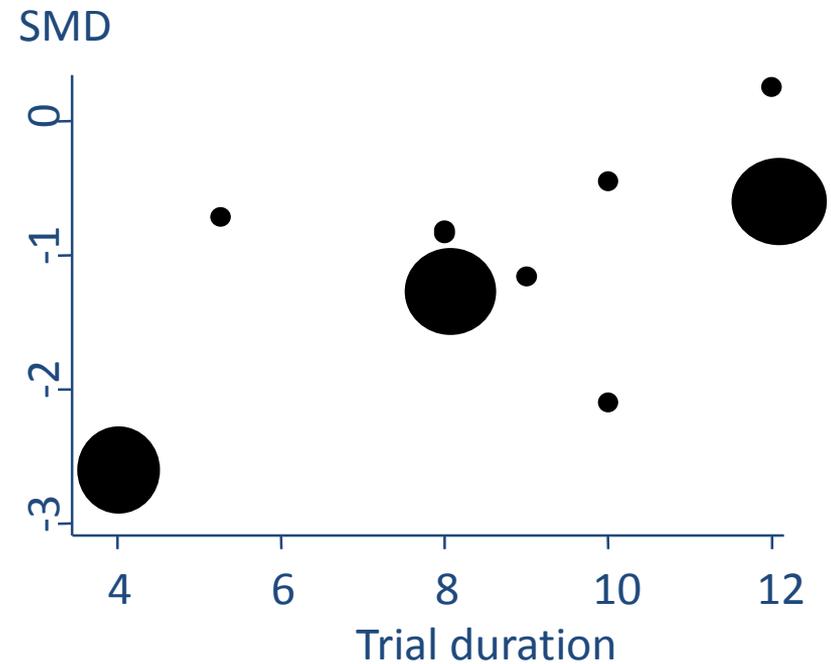


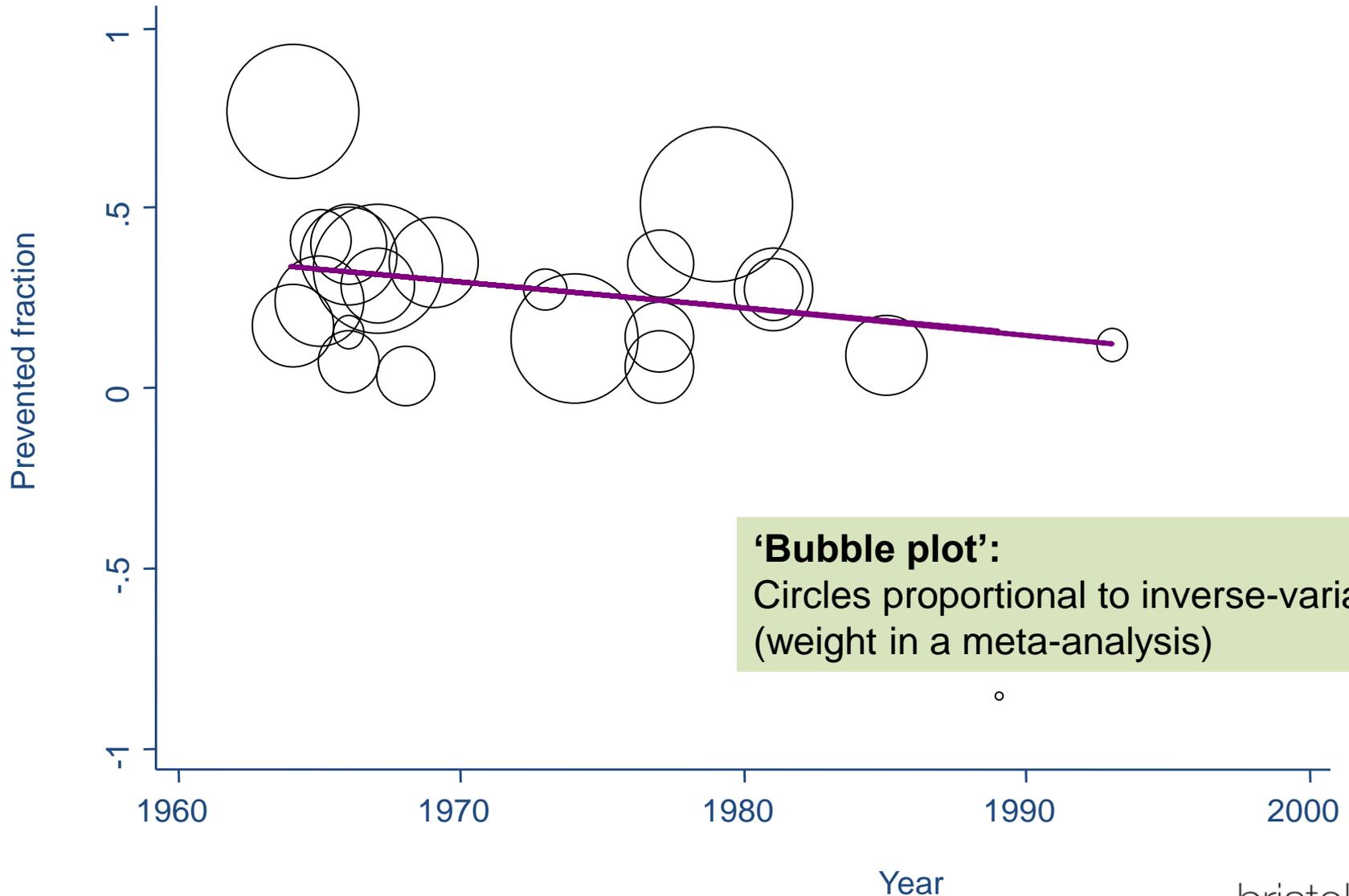
Why we don't use simple linear regression

- Just as in meta-analysis, the studies are different sizes, and should have different influences on the analysis
- There's a difference between

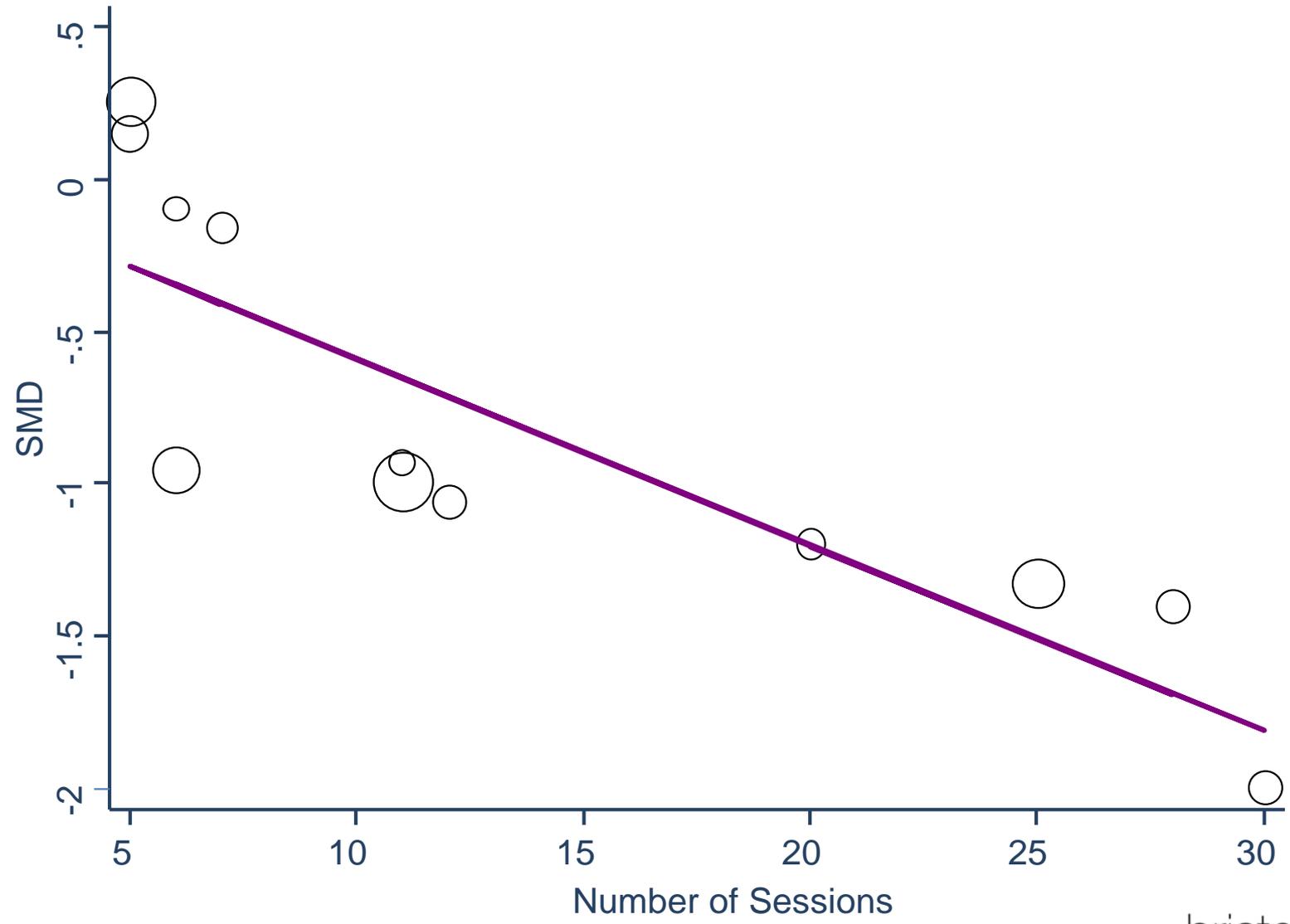


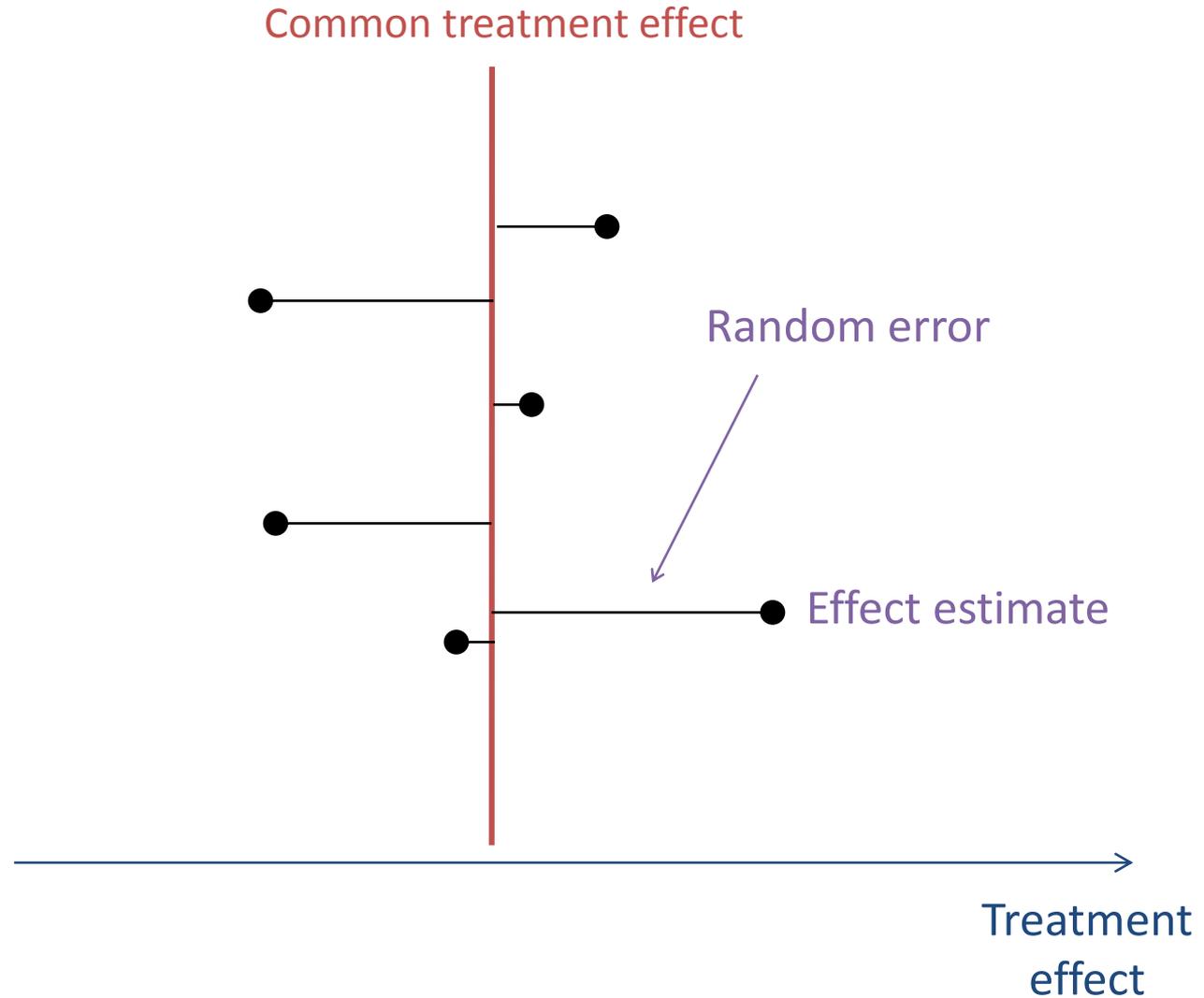
and





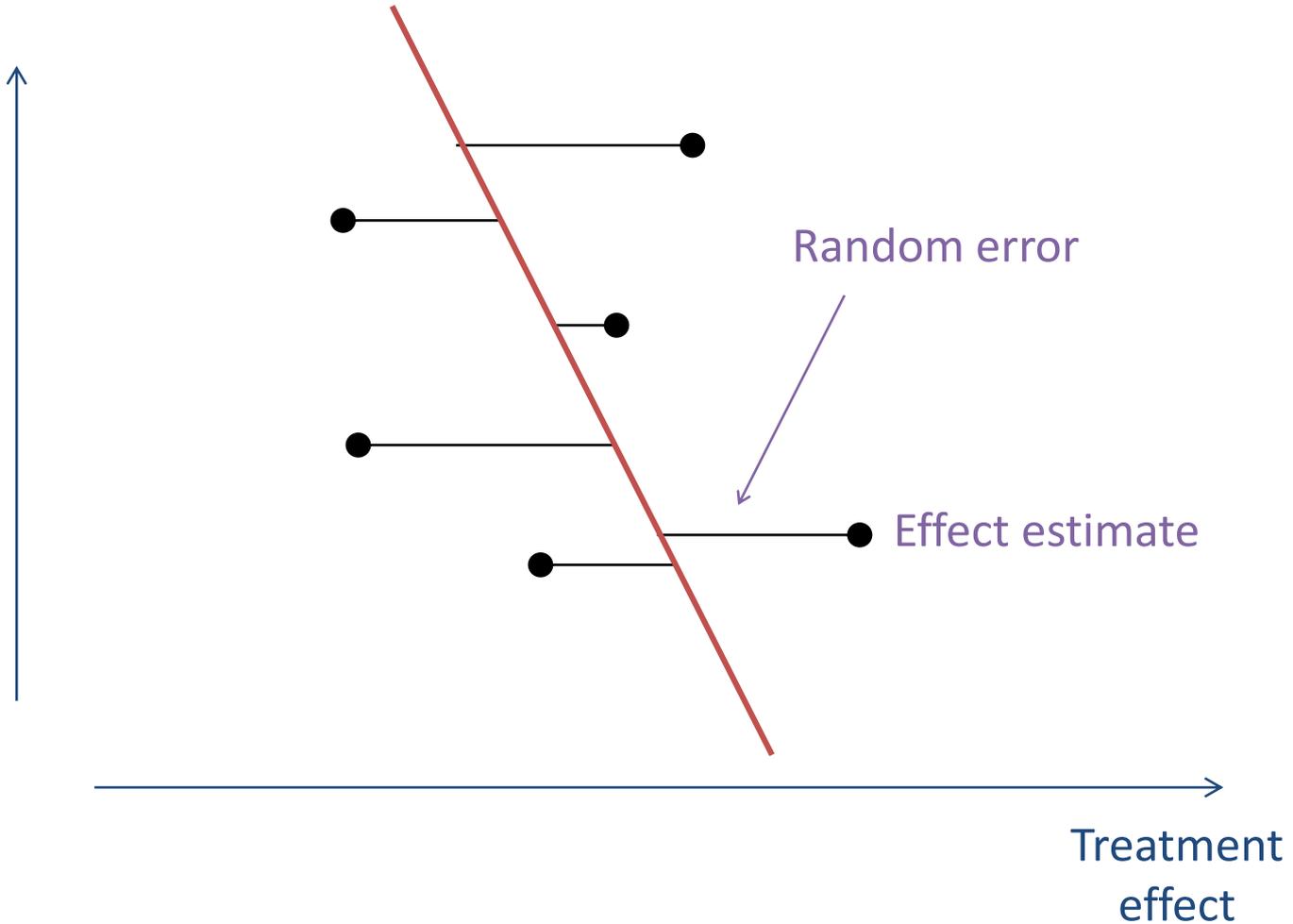
Number of sessions of CBT



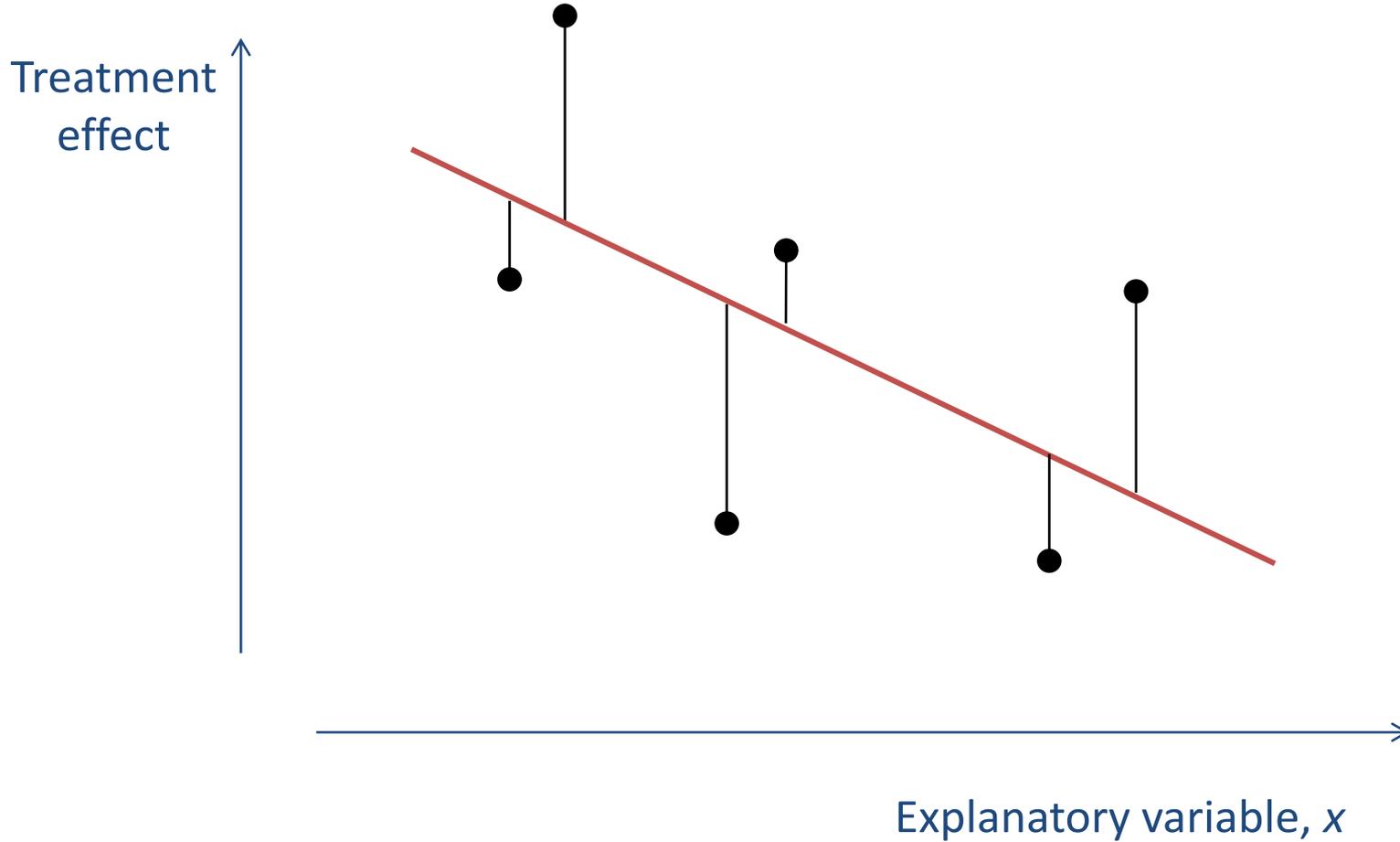


Explanatory
variable, x

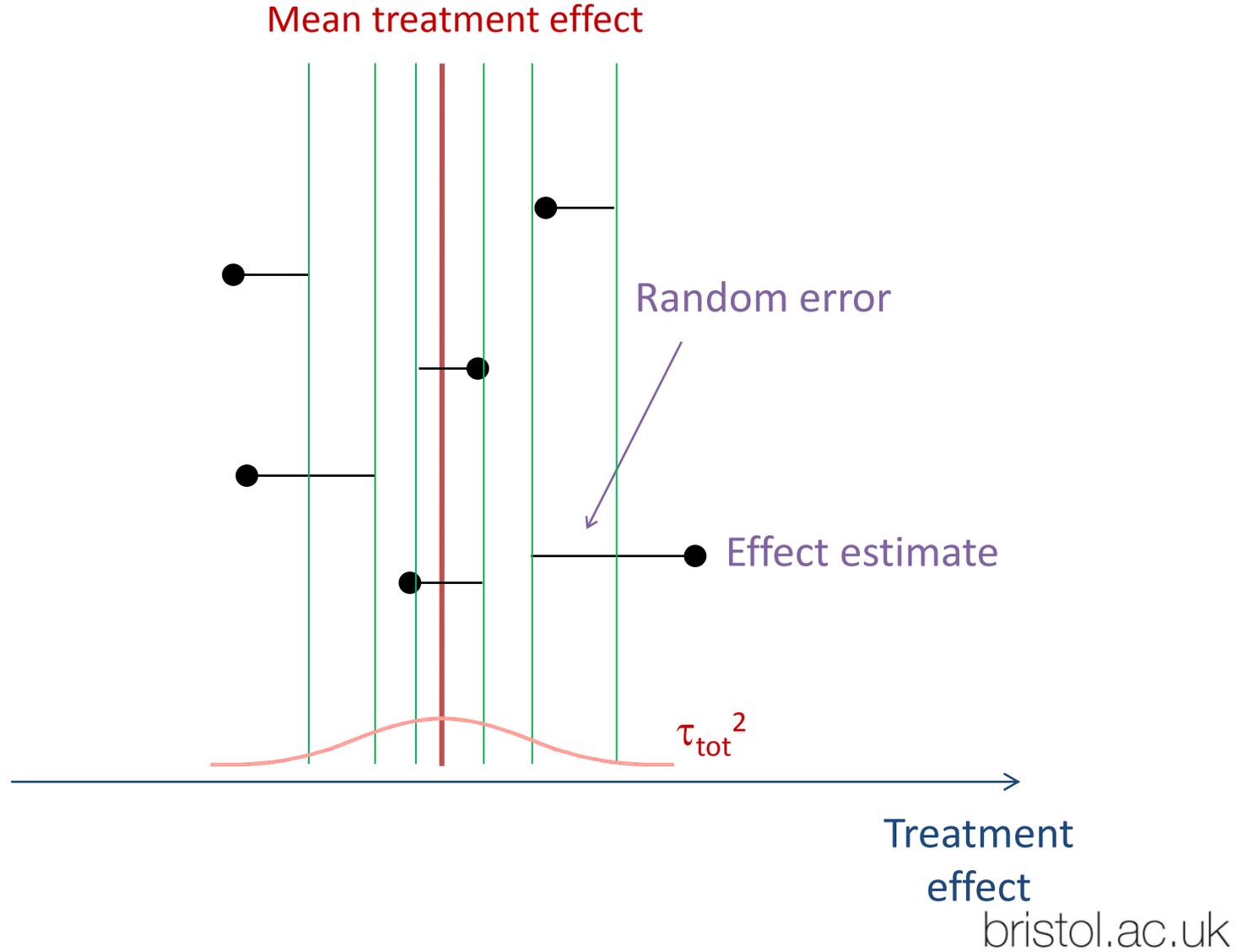
Treatment effect = intercept + slope $\times x$



The other way round (the convention for plotting)

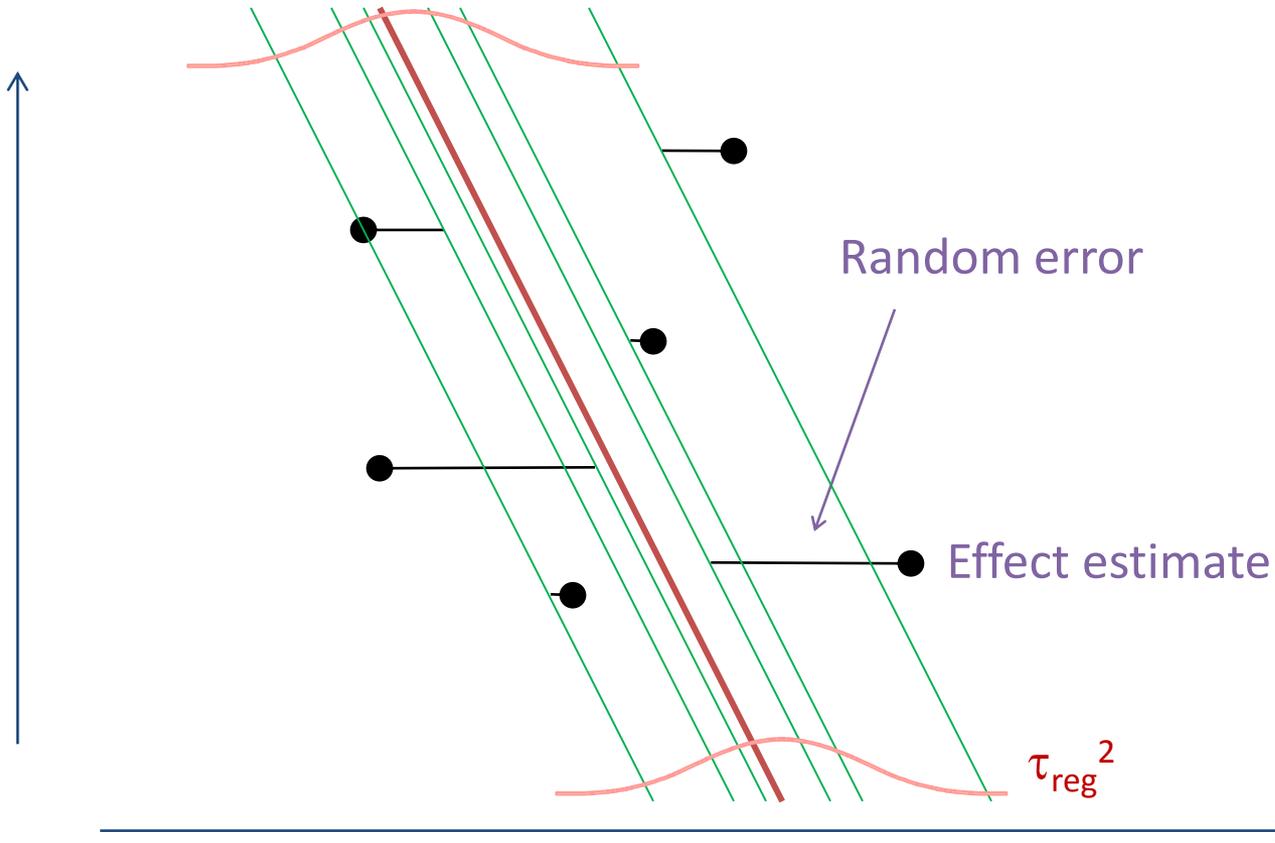


Random-effects meta-analysis



Mean treatment effect = intercept + slope $\times x$

Explanatory
variable, x



Treatment
effect

- Compare
heterogeneity variance from random-effects
meta-analysis (τ_{tot}^2)
with
heterogeneity variance from random-effects
meta-regression (τ_{reg}^2)
- % variance explained = $100\% \times \frac{\tau_{tot}^2 - \tau_{reg}^2}{\tau_{tot}^2}$
- A useful measure of the explanatory ability of a (set of)
covariate(s)

It has been recognised for many years that the protection given by BCG against tuberculosis varied between trials

Risk ratios:

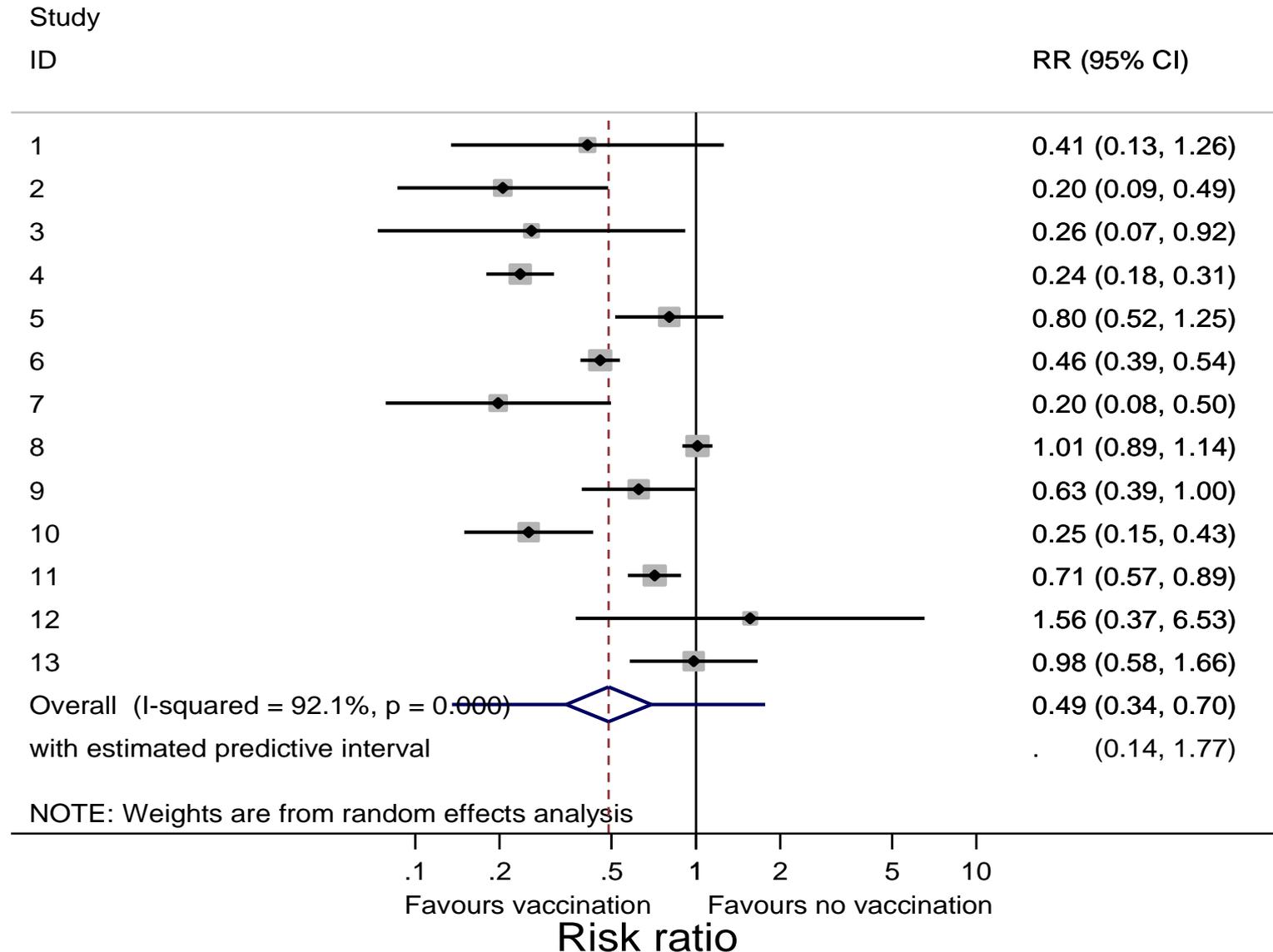
MRC trial (UK)	0.24 (95% CI 0.18 to 0.31)
Madras, (South India)	1.01 (95% CI 0.89 to 1.14)

Random-effects meta-analysis (Colditz et al. 1994)

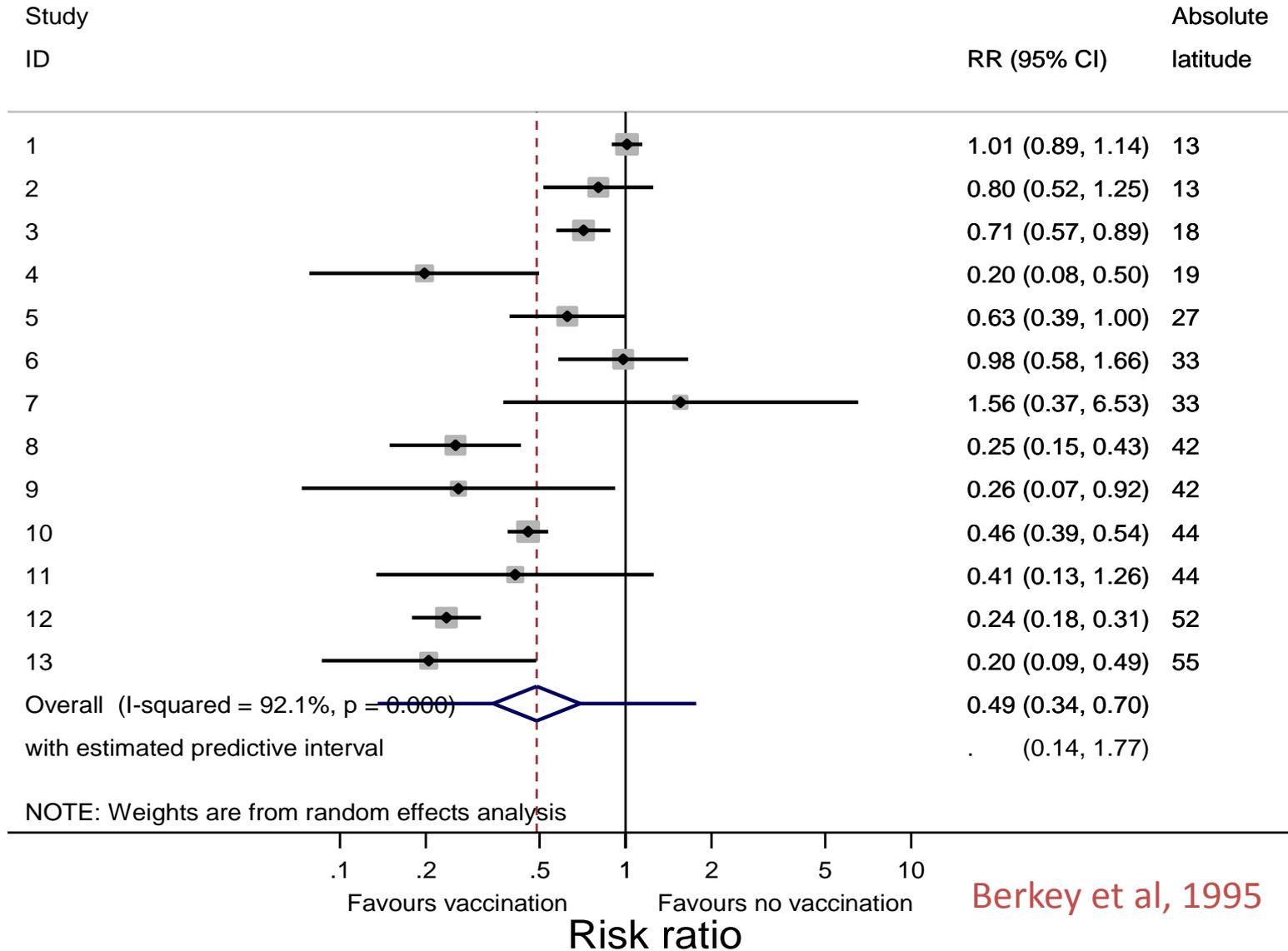
Summary (random effects) RR: 0.49 (95% CI 0.34 to 0.70)

“the results of this meta-analysis lend added weight and confidence to arguments favouring the use of BCG vaccine”

BCG vaccination to prevent tuberculosis

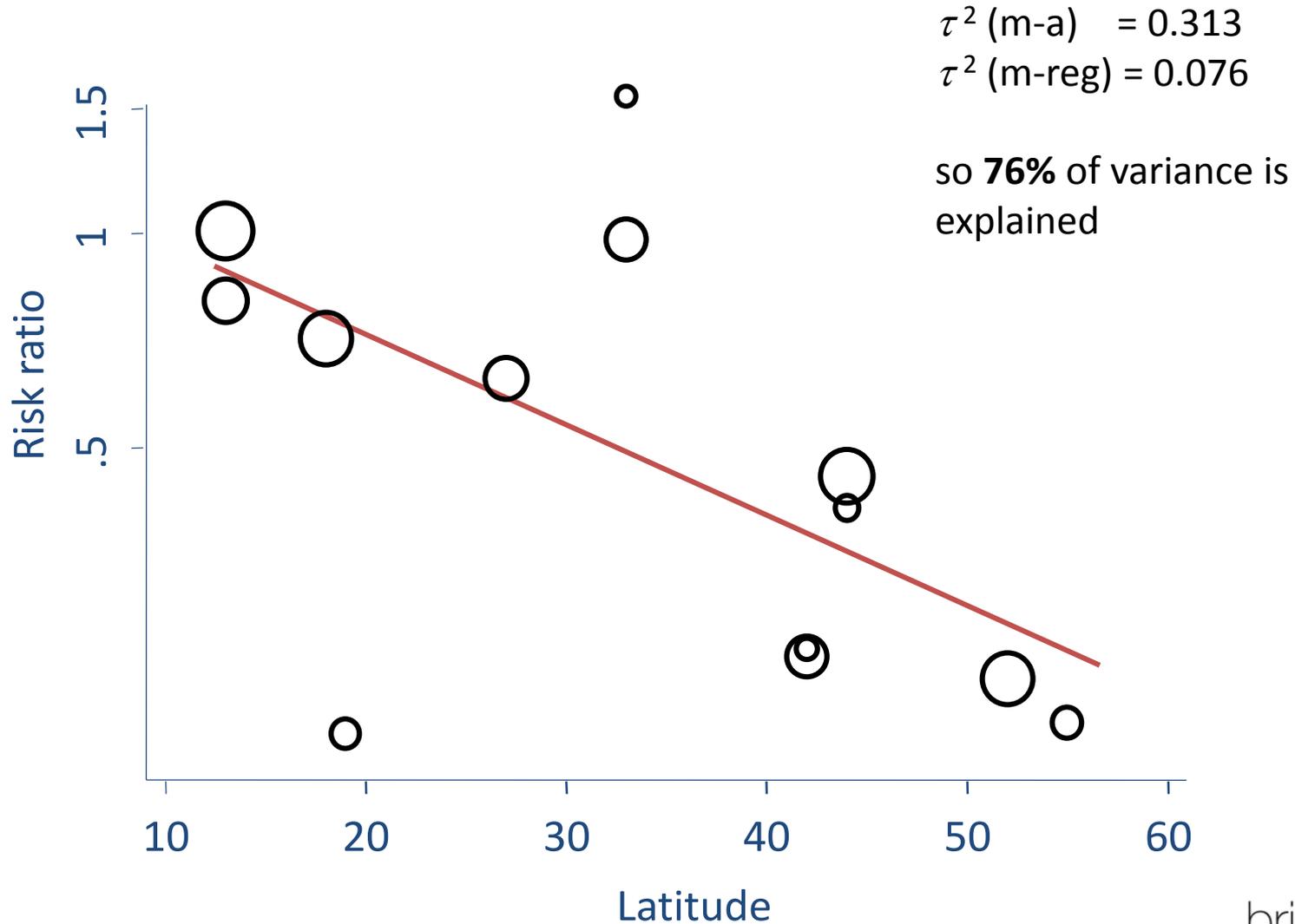


BCG vaccination to prevent tuberculosis



Berkey et al, 1995

Meta-regression: Dependence of BCG vaccine efficacy on study latitude



- Simulation study:

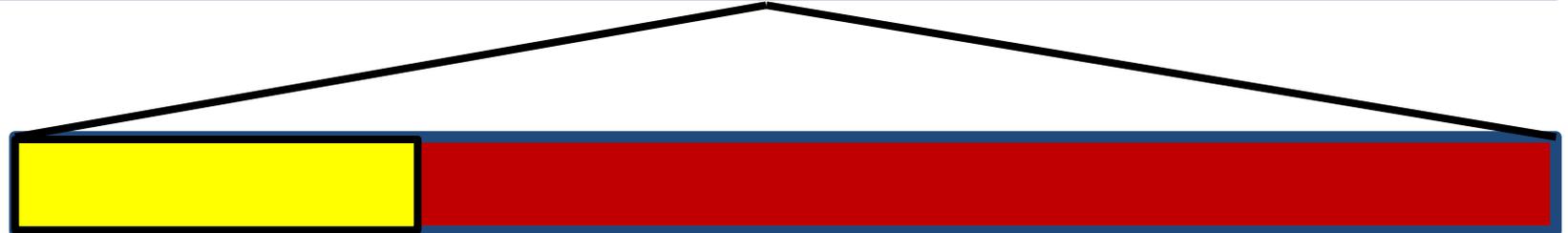
“Of the different methods evaluated, only the Knapp and Hartung method and the permutation test provide adequate control of the Type I error rate across all conditions. Due to its computational simplicity, the Knapp and Hartung method is recommended as a suitable option for most meta-analyses.”

Viechtbauer et al 2015

- More on permutation tests later

Within
studies:
8%

Between studies
(I^2):
92%

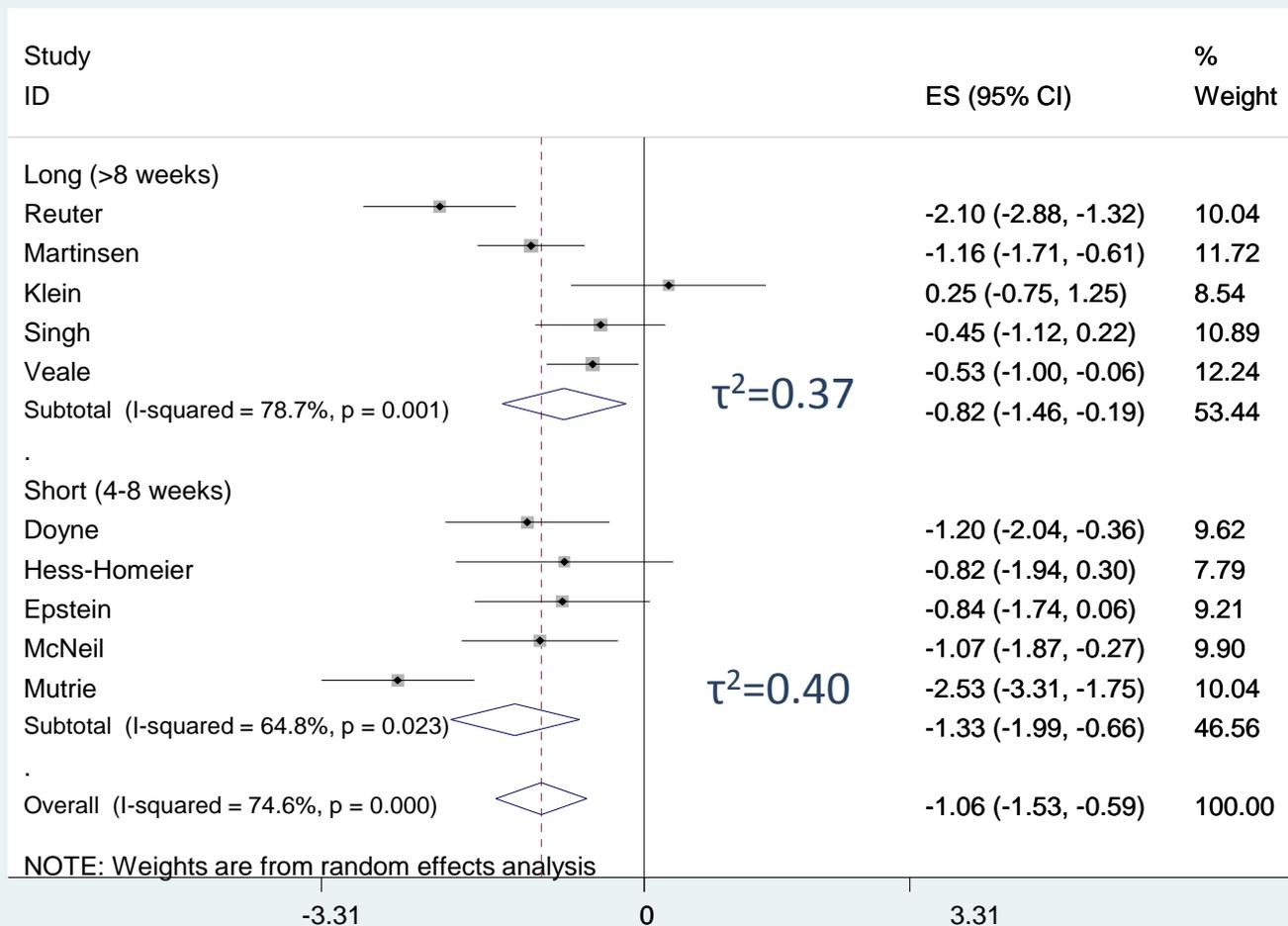


Unexplained:
24%

Explained by latitude (R^2):
76%

- The key difference is the assumption made about heterogeneity in random-effects analyses
 - estimated separately for each subgroup in subgroup analyses
 - (as usually implemented)
 - assumed equal across subgroups in meta-regression
 - (as usually implemented)
- Of course, there is flexibility to do things in different ways
 - some software assumes equal heterogeneity when performing subgroup analyses

```
. metan smd sesmd, random by(duration) label(namevar=study)
```



Tests for subgroup differences

1. Q test from subgroup results:
P = 0.28
(RevMan/metan)

2. Meta-regression:
P = 0.34
(metareg)

```
. metareg smd duration, wsse(sesmd) mm
```

Meta-regression

Number of studies = 10

Method of moments estimate of between-study variance: tau2 = .3893

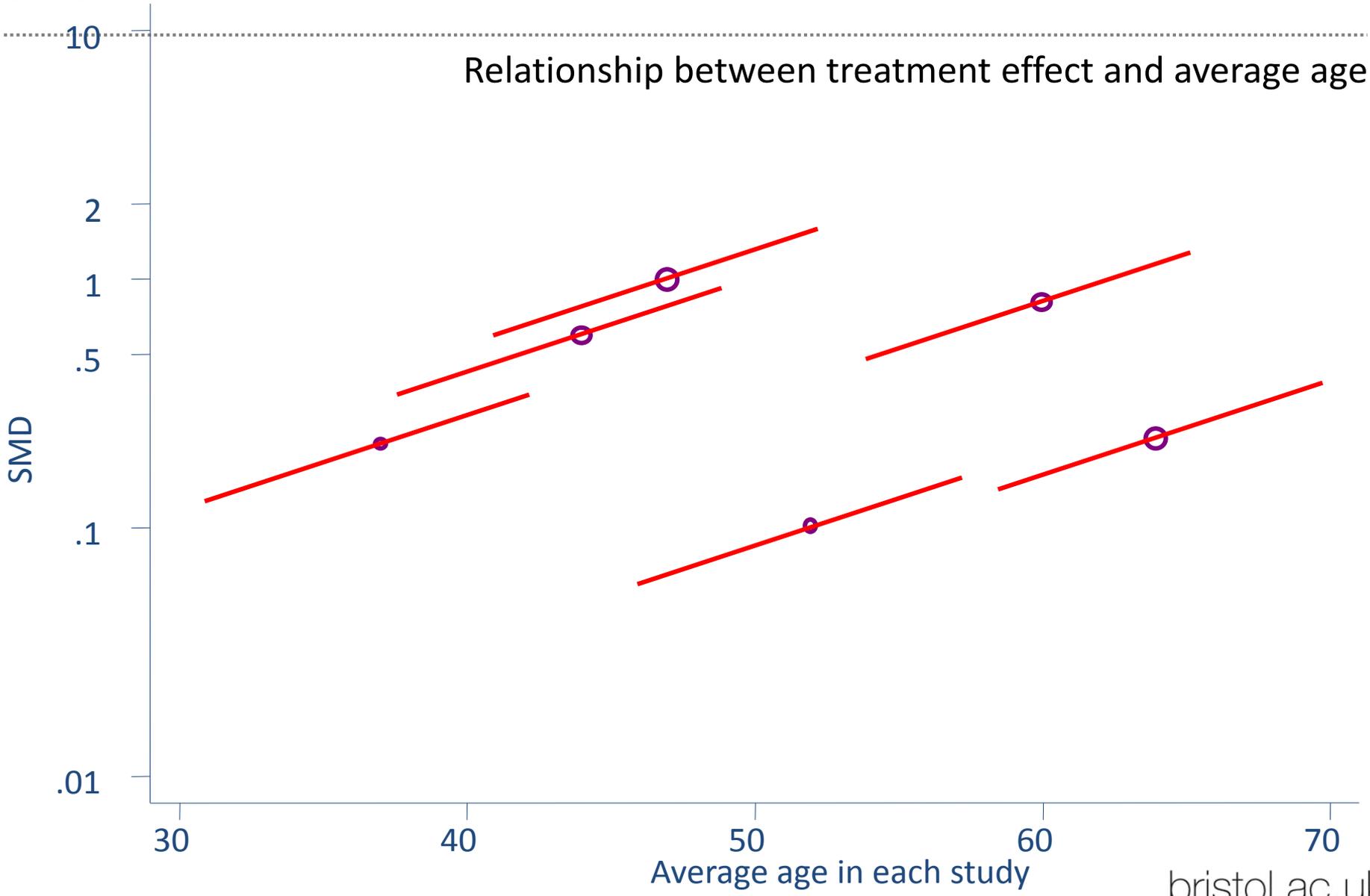
smd	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
duration	-.5026523	.4990484	-1.01	0.343	-1.65346 .6481553

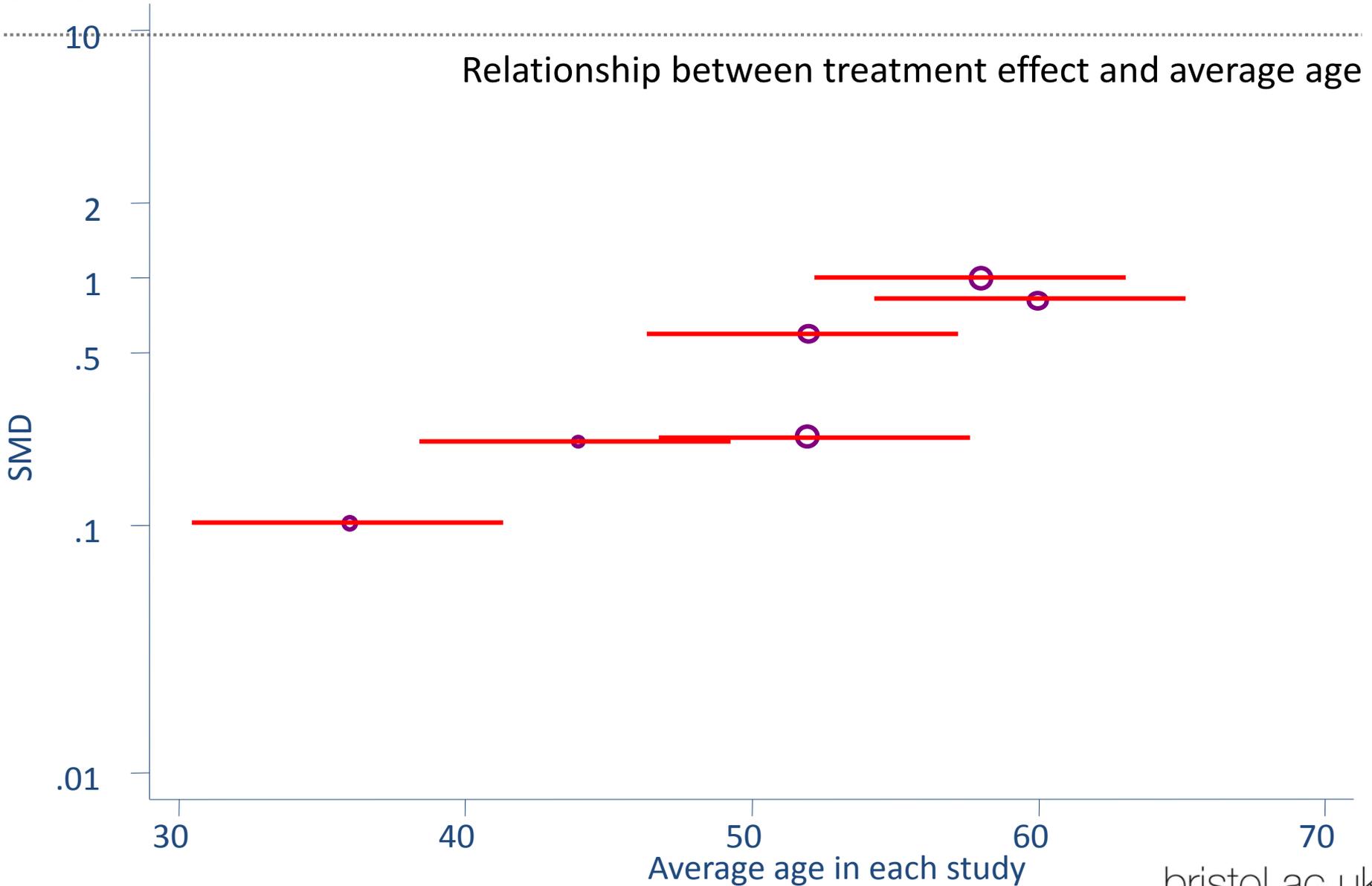
	Continuous outcome (MD/SMD)	Dichotomous outcome (OR/RR)
Continuous explanatory variable	Difference in mean differences or in SMDs <i>associated with 1 point increase in covariate</i>	Ratio of ORs or of RRs <i>associated with 1 point increase in covariate</i>
Dichotomous explanatory variable	Difference in mean differences or in SMDs <i>between 2 groups</i>	Ratio of ORs or of RRs <i>between 2 groups</i>

Meta-regression with dichotomous variable gives formal comparison between subgroups

Part 4: Problems

- Beware study characteristics that summarize participants within a study, e.g.
 - average age
 - % females
 - average duration of follow-up
 - % drop out





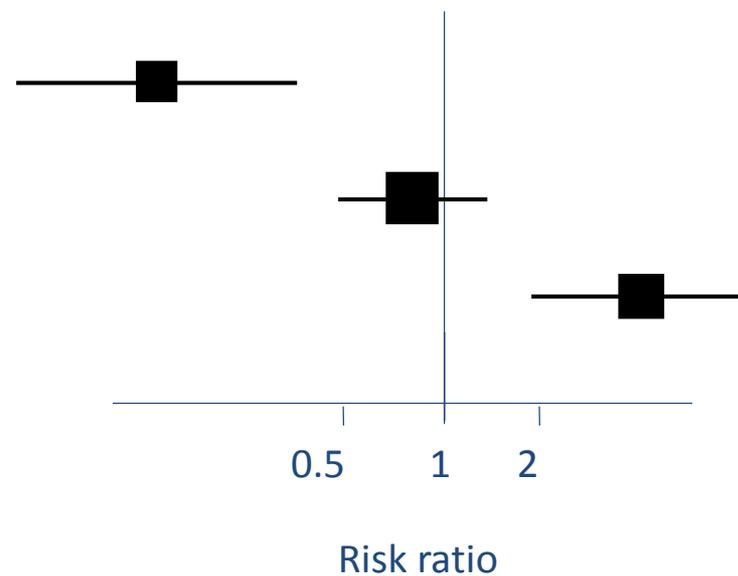
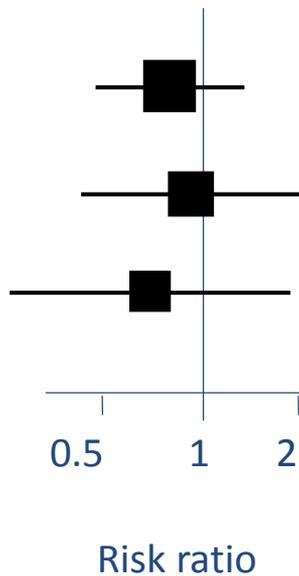
-
- Beware study characteristics that summarize participants within a study, e.g.
 - average age
 - % females
 - average duration of follow-up
 - % drop out
 - Relationships **across** studies may not reflect relationships **within** studies
 - The relationship between treatment effect and age, sex, etc is best measured within a study
 - Needs individual participant data

- Unfortunately most meta-analyses in Cochrane reviews do not have many studies
- Meta-regression typically has **low power** to detect relationships
- Model diagnostics / adequacy difficult to assess

- There are typically many, many explanatory variables to choose from
 - Heterogeneity can always be explained if you look at enough of them
 - Great risk of spurious findings

- To guard against false-positives, meta-analysts are advised to
 - Pre-specify characteristics in a protocol
 - Limit to a small number
 - Have a scientific rationale
- Beware ‘prognostic factors’
 - Variables that predict clinical outcome don’t necessarily affect treatment effects
 - e.g. age may be strongly prognostic, but risk ratios may well be the same irrespective of age

- Depends on
 - number of studies
 - extent of heterogeneity
 - precision of effect estimates



Ave. age

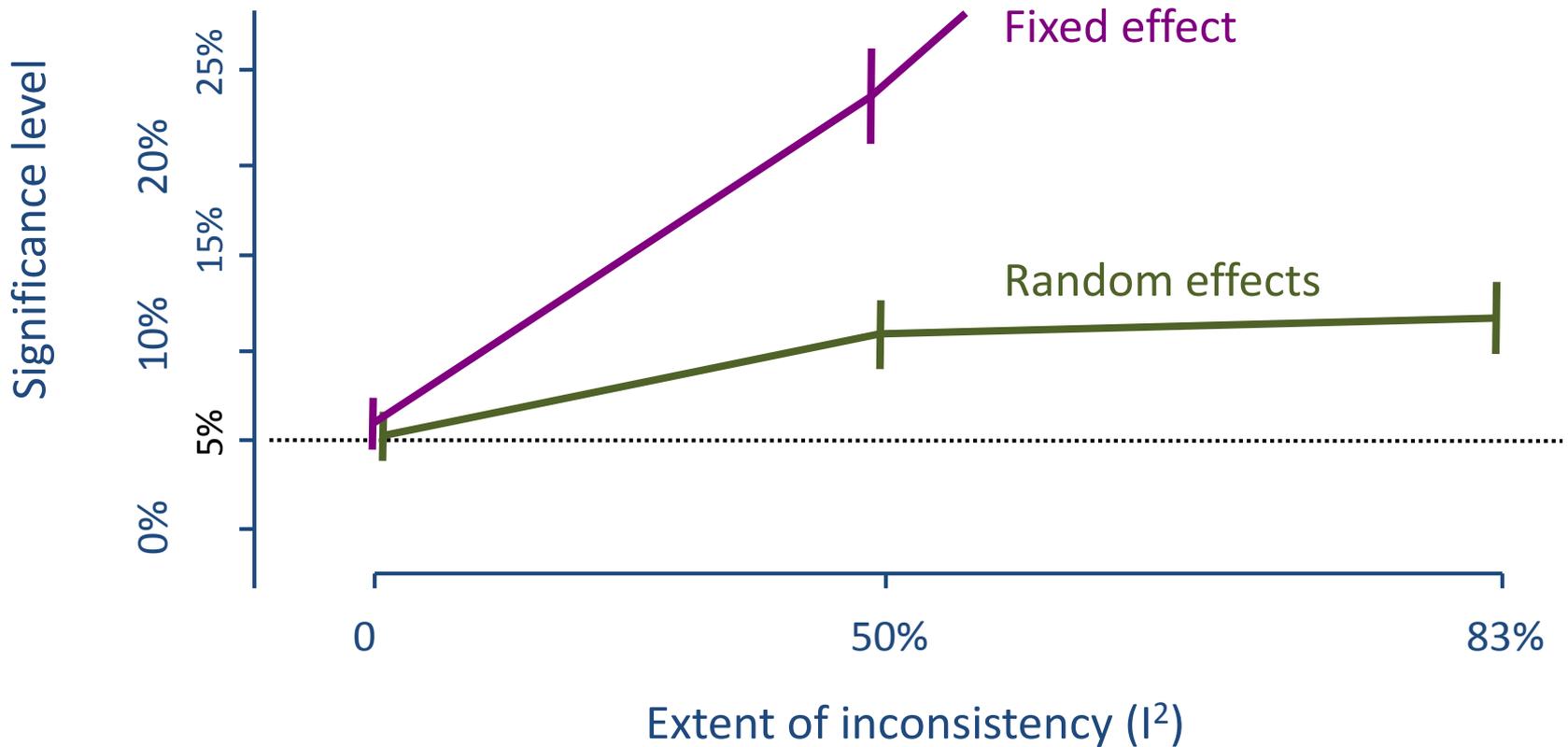
8 wks

16 wks

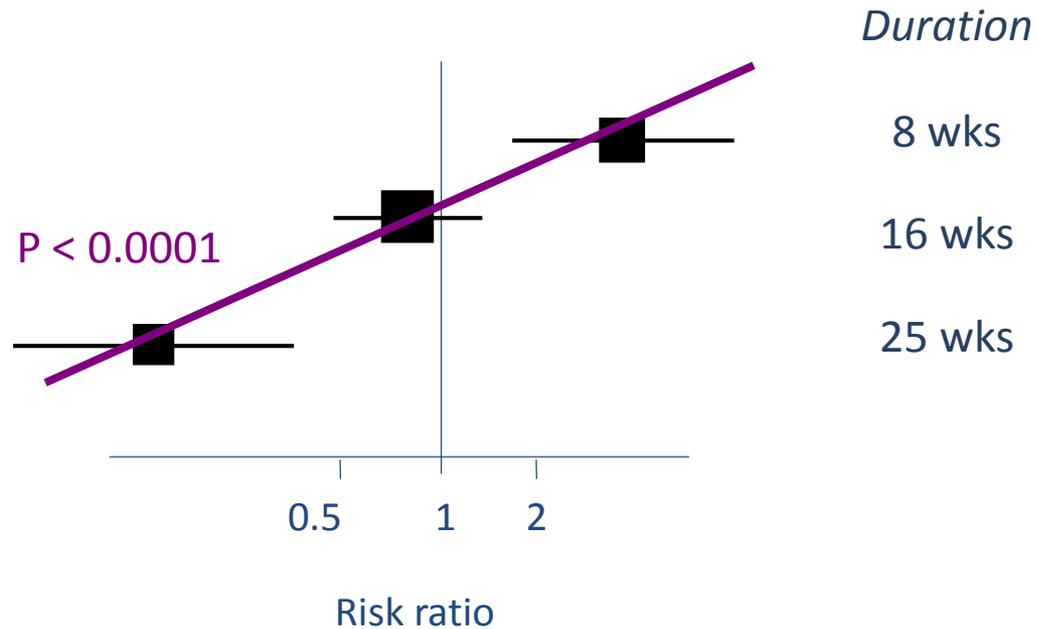
24 wks

Simulation study of false-positive rates

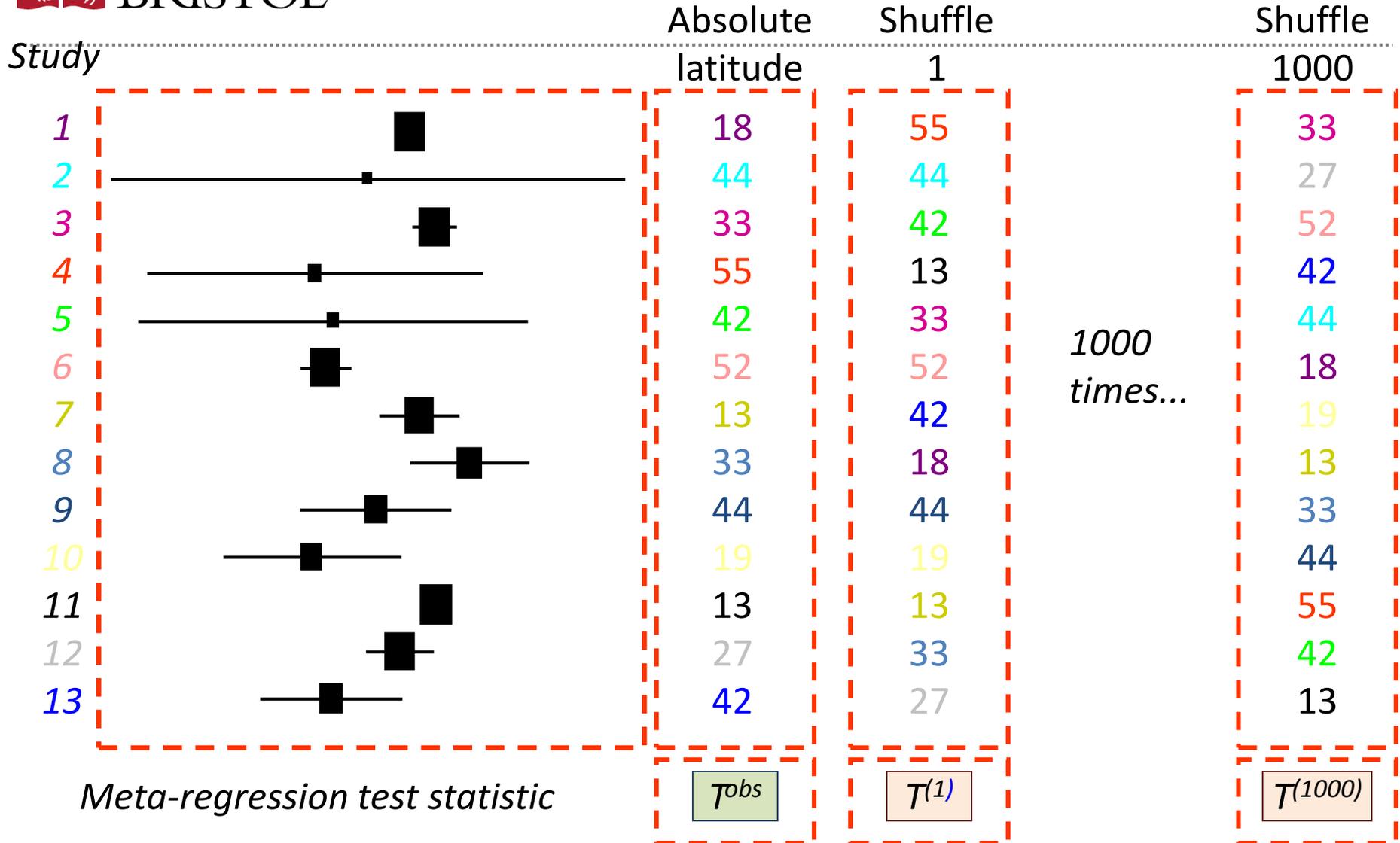
Example results: 10 studies, typical study weights, 1 covariate



Solution? A permutation test

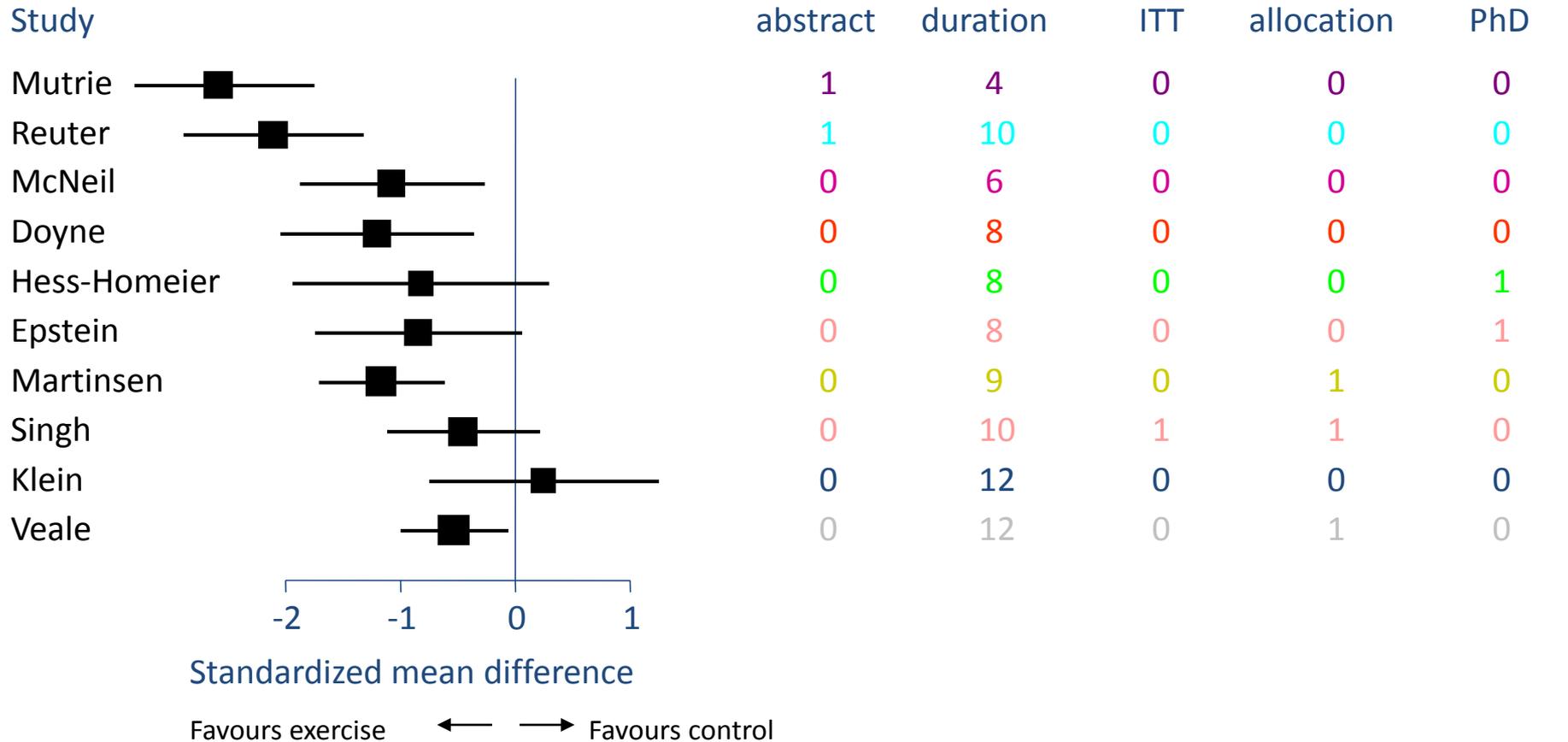


- But 1/3 of possible permutations give a relationship *at least as strong* as this
- So 'real' $p = 0.33$



- Compare T^{obs} with the values $T^{(1)} \dots , T^{(1000)}$
- Count the number of T 's that exceed T^{obs}
- e.g. for BCG example $T^{obs} = 2.44$ (= slope/SE);
- 54/1000 T 's exceed T^{obs} : $p = 0.054$
- c.f.
 - FE: $p < 10^{-10}$
 - RE (standard normal): $p = 0.012$

Multiple testing example: Exercise for depression



RE meta-regression (STATA)

p=0.0006

p=0.01

p=0.24

p=0.25

p=0.98

Perm. test for "nth" most significant

p=0.11

p=0.09

p=0.25

p=0.10

p=0.88

Part 5: Closing remarks

Selecting explanatory variables

Meta-regression in Cochrane reviews

Extensions

Summary

- Specify a **small number** of characteristics **in advance**
- Ensure there is scientific rationale for investigating each characteristic
 - **effect modifier or prognostic factor?**
- Make sure the effect of a characteristic can be identified
 - **does it differentiate studies?**
 - **aggregation bias**
- Think about whether the characteristic is closely related to another characteristic
 - **confounding**

How many studies / characteristics?

- Typical guidance is to have 10 studies for each characteristic examined
- Some say 5 studies is enough

- Authors are encouraged to use meta-regression if appropriate
- ‘Bubble’ plots may be included as an ‘Other figure’
- Results should be presented in an ‘Additional table’
- Consider presenting something like:

Explanatory variable	Slope or Exp(slope)	95% confidence interval	Proportion of variation explained	Interpretation
Duration	OR = 1.3	0.9 to 1.8	14%	Odds ratio increases with duration (not statistically significant)

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- **Baseline risk** of the studied population (as measured in the control group) might be considered as an explanatory variable
 - Beware! It is inherently correlated with treatment effects
 - Special methods are needed (see Thompson et al 1997)
 - **Precision of the estimate (e.g. its standard error)** might be considered as an explanatory variable to look at small study effects (may be suggestive of publication bias)
 - Beware! It is often inherently correlated with treatment effects
 - Special methods are needed (see Sterne et al 2011)

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- Subgroup analysis and meta-regression examine the relationship between treatment effects and one or more study-level characteristics
 - Meta-regression is like usual linear regression, but
 - study weights incorporated
 - variation not explained by the characteristics should be accounted for
 - random effects meta-regression does this; fixed effect meta-regression does not

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- Subgroup analyses are meta-regression are great in theory, and easy to perform in STATA or R (and other software)
 - They are fraught with dangers and should be undertaken and interpreted with caution
 - observational relationships
 - too few studies
 - too many sources of diversity and bias
 - confounding and aggregation bias

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